

**THE CUMULATIVE DISTRIBUTION FUNCTION OF THE
DOUBLY-TRUNCATED NORMAL DISTRIBUTION**

Arvid C. Johnson, Dominican University, River Forest, IL

**Definition of the Standardized, Doubly-Truncated Normal
Distribution**

Consider a normally-distributed random variable x with a probability density function $f(x)$ specified as

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, -\infty \leq x \leq \infty \quad (1)$$

If the values of x below some value x_L and above some value x_R cannot be observed - due to censoring or truncation - then, as shown in Figure 1, the resulting distribution is a doubly-truncated normal distribution with probability density function $f_{DTN}(x)$ given by

$$f_{DTN}(x) = \begin{cases} 0, & -\infty \leq x \leq x_L \\ \frac{f(x)}{\int_{x_L}^{x_R} f(x)dx}, & x_L \leq x \leq x_R \\ 0, & x_R \leq x \leq \infty \end{cases} \quad (2)$$

where $f(x)$ is as defined in Equation 1.

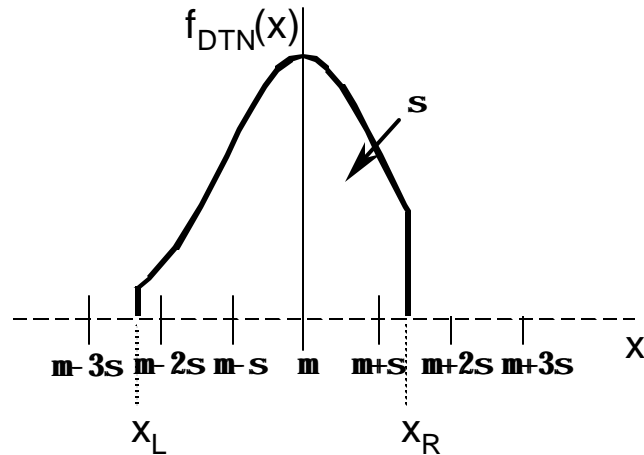


Figure 1. Doubly-Truncated Normal Distribution (in Terms of the Original Population Parameters)

For purposes of generality, Equation 2 can be re-stated in terms of the standard normal distribution (denoted $f(z)$) where

$$z = \frac{x - \mu}{\sigma} \quad (3)$$

and

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}, -\infty \leq z \leq \infty \quad (4)$$

In terms of this standard normal distribution, the points of truncation x_L and x_R will be denoted by k_L and k_R , respectively, as given by

$$k_L = \frac{x_L - \mu}{\sigma} \quad \text{and} \quad k_R = \frac{x_R - \mu}{\sigma} \quad (5)$$

Reformulating the doubly-truncated normal distribution of Equation 2 in terms of the standard normal distribution, the following can be found:

$$f_{\text{DTN}}(z) = \begin{cases} 0, & -\infty \leq z \leq k_L \\ \frac{f(z)}{\int_{k_L}^{k_R} f(z) dz}, & k_L \leq z \leq k_R \\ 0, & k_R \leq z \leq \infty \end{cases} \quad (6)$$

This is illustrated in Figure 2.

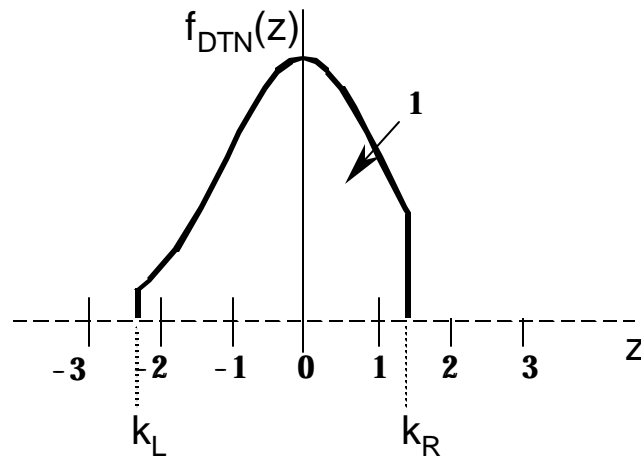


Figure 2. Doubly-Truncated Normal Distribution (in Terms of the Standard Normal Distribution)

To define what this paper terms as a "standardized, doubly-truncated normal distribution," introduce a standardizing variable $t = z - k_L$, which has the effect of defining the left-most point of truncation as $t = 0$. The standardized, doubly-truncated normal distribution $f_{\text{SDTN}}(t)$ is, thus, given by

$$f_{\text{SDTN}}(t) = \begin{cases} 0, & t \leq 0 \\ \frac{f(t + k_L)}{\int_{k_L}^{k_R} f(z) dz}, & 0 \leq t \leq t_R \\ 0, & t_R \leq t \end{cases}, \quad (7)$$

and is illustrated in Figure 3.

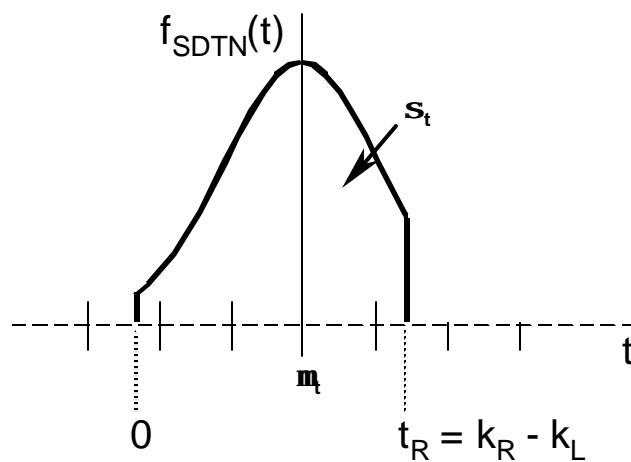


Figure 3. Standardized, Doubly-Truncated Normal Distribution

It should be noted that, without loss of generality, by defining the lower truncation point k_L as $t = 0$, the

upper truncation point k_R has a standardized t value of $t_R = k_R - k_L$.

This paper, next, develops formulas for the two parameters of the standardized, doubly-truncated normal distribution: the mean (μ_t) and the standard deviation (σ_t) - for given points of truncation k_L and k_R .

Parameters of the Standardized, Doubly-Truncated Normal Distribution

Consider the mean (μ_t) of the standardized, doubly-truncated normal distribution - for given points of truncation k_L and k_R , where $f(t)$ is as defined in Equation 7.

$$\mu_t = \mathbb{E}(t) = \int_0^{t_R} t f(t) dt \quad (8)$$

With $t = z - k_L$, it follows that $dt = dz$, $z = k_L$ for $t = 0$, and $z = k_R$ for $t = t_R$. Using this, Equation 8 becomes

$$\begin{aligned} \mu_t &= \frac{1}{\int_{k_L}^{k_R} f(z) dz} \int_{k_L}^{k_R} (z - k_L) f(z) dz \\ &= \frac{1}{\int_{k_L}^{k_R} f(z) dz} \left[\int_{k_L}^{k_R} z f(z) dz - k_L \int_{k_L}^{k_R} f(z) dz \right] \end{aligned} \quad (9)$$

Consider the first term within the brackets in Equation 9. To evaluate this term, one can make use of the identity [Bey87]

$$\int x e^{-x^2} dx = -\frac{1}{2} e^{-x^2} \quad (10)$$

Using Equations 4 and 10, it can be shown that

$$\begin{aligned}
\int_{k_L}^{k_R} z f(z) dz &= \int_{k_L}^{\infty} z f(z) dz - \int_{k_R}^{\infty} z f(z) dz = \frac{1}{\sqrt{2\pi}} \int_{k_L}^{\infty} z e^{-\frac{1}{2}z^2} dz - \frac{1}{\sqrt{2\pi}} \int_{k_R}^{\infty} z e^{-\frac{1}{2}z^2} dz \\
&= -\frac{1}{\sqrt{2\pi}} e^{-x^2} \Big|_{k_L/\sqrt{2}}^{\infty} + \frac{1}{\sqrt{2\pi}} e^{-x^2} \Big|_{k_R/\sqrt{2}}^{\infty} \\
&= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}k_L^2} - \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}k_R^2} \\
&= f(k_L) - f(k_R)
\end{aligned} \tag{11}$$

Defining $F(k)$ as

$$F(k) = \int_{-\infty}^k f(z) dz \tag{12}$$

and using Equations 11 and 12, Equation 9 can be re-stated in its final form as

$$\mu_t = \frac{1}{F(k_R) - F(k_L)} \left[\{f(k_L) - f(k_R)\} - k_L \{F(k_R) - F(k_L)\} \right] \tag{13}$$

From Equation 5-13, it is clear that the mean of the standardized, doubly-truncated normal distribution is solely dependent upon the points of truncation k_L and k_R .

Consider now the standard deviation (σ_t) of the standardized, doubly-truncated normal distribution. Given that [Hin90]

$$\sigma_t^2 = E(t^2) - \mu_t^2 \tag{14}$$

It is only necessary to find $E(t^2)$.

$$\begin{aligned} E(t^2) &= \int_0^{t_R} t^2 f(t) dt = \frac{1}{F(k_R) - F(k_L)} \int_{k_L}^{k_R} (z - k_L)^2 f(z) dz \\ &= \frac{1}{F(k_R) - F(k_L)} \left[\int_{k_L}^{k_R} z^2 f(z) dz - 2k_L \int_{k_L}^{k_R} z f(z) dz + k_L^2 \int_{k_L}^{k_R} f(z) dz \right] \end{aligned} \quad (15)$$

The last two terms within the brackets in Equation 15 have already been encountered in Equations 11 and 12, respectively. Consider, now, the first term - which, via integration by parts, can be found to be equivalent to

$$\int_{k_L}^{k_R} z^2 f(z) dz = -z f(z) \Big|_{k_L}^{k_R} + \int_{k_L}^{k_R} f(z) dz = -k_R f(k_R) + k_L f(k_L) + F(k_R) - F(k_L) \quad (16)$$

Utilizing Equations 11, 12, and 16, Equation 15 can be restated as

$$\begin{aligned} E(t^2) &= \frac{1}{F(k_R) - F(k_L)} \left[\begin{array}{l} -k_R f(k_R) + k_L f(k_L) + F(k_R) - F(k_L) \\ -2k_L (f(k_L) - f(k_R)) + k_L^2 (F(k_R) - F(k_L)) \end{array} \right] \\ &= \frac{1}{F(k_R) - F(k_L)} \left[\{2k_L - k_R\} f(k_R) - k_L f(k_L) + (1 + k_L^2) \{F(k_R) - F(k_L)\} \right] \end{aligned} \quad (17)$$

Equation 17 can be used along with Equation 13 to calculate σ_t as shown in Equation 14. Once again, it is worth noting that the standard deviation of the

standardized, doubly-truncated normal distribution is solely dependent upon the points of truncation k_L and k_R .

Finally, it is convenient to define a coefficient of variation c - which, again, is determined solely by the particular values of k_L and k_R

$$c = \frac{\sigma_t}{\mu_t} \tag{18}$$

**Development of Tables of the Cumulative Distribution
Function of the Standardized, Doubly-Truncated Normal
Distribution**

Given the formulation of the probability density function $f_{SDTN}(t)$ of the standardized, doubly-truncated normal distribution in Equation 7, its cumulative distribution function $F_{SDTN}(t)$ can be stated as

$$F_{SDTN}(t) = \begin{cases} 0, & t \leq 0 \\ \frac{F(t + k_L) - F(k_L)}{F(k_R) - F(k_L)} & 0 \leq t \leq t_R \\ 1, & t_R \leq t \end{cases} \quad (19)$$

where $F(k)$ is as defined in Equation 12 and $F(z)$ is the cumulative probability associated with a standard normal variate of value z .

Thus, given points of truncation k_L and k_R , one can readily calculate the value of the cumulative distribution function of the standardized, doubly-truncated normal distribution at any standardized value of $t \geq 0$ through the use of Equation 19 and tables of the cumulative distribution function of the standard normal distribution $F(z)$.

With the wide availability of desktop micro-computers and their associated software, alternate methods of evaluating Equation 19 at specific values of t include the use of "standard" spreadsheet functions - e.g., NormSDist(Z) in Microsoft Excel - or formulations such as

those provided in [Abr72]. In developing the tables presented below, the results of utilizing the Excel functions were compared with [Abr72]'s smallest error formulation: ($|\varepsilon(x)| < 7.5 \times 10^{-8}$)

$$F(x) = 1 - \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} (b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5)$$

where

$$t = \frac{1}{1 + px} \tag{20}$$

$p = 0.231619, b_1 = 0.319381530, b_2 = -0.356563782$
 $b_3 = 1.781477937, b_4 = -1.821255978, b_5 = 1.330274429$

In general, the results of the Excel-based calculations and those based upon Equation 20 were found to be close enough that either could be used for purposes of developing the tables presented in this paper. Specifically, the values of the cumulative distribution calculated using the Excel worksheet functions agreed with those tabulated in [Abr72] to at least six decimal places. Even after computations involved with one of the most heavily truncated cases presented in this paper (i.e., $k_L = 2.0$ and $k_R = 3.0$), Excel's results agreed with those obtained using the tabulated values in [Abr72] to at least four decimal places. The Excel-based approach has been utilized in this paper due to its ease of implementation - with the results/values being shown to four decimal places.

Using the approach outlined above, a table of the standardized value t at which the cumulative distribution function $F_{SDTN}(t)$ of the standardized, doubly-truncated normal distribution function assumes some value has been developed. This problem was solved computationally through the use of a dichotomous line search algorithm, and the results are presented in Table 1 over the range $F_{SDTN}(t) = 0.01$ to $F_{SDTN}(t) = 0.99$, with varying granularity. Table 1 is presented in thirteen segments - for allowable combinations of k_L and k_R over the range $-3.0(0.2)3.0$. ("Allowable combinations" simply means that the upper truncation point k_R must be greater than the lower truncation point k_L .)

Review of Table 1 indicates that, unlike the standardized, left- and right-truncated normal distributions, the standardized, doubly-truncated normal distribution does not have a uniquely-determined coefficient of variation c . This is summarized in Table 2 - which lists the mean, standard deviation, and coefficient of variation for allowable combinations of the lower and upper truncation points. Table 2 is presented in five segments grouped by lower truncation point k_L .

Table 1. Standard t Value Associated with a Cumulative Distribution Value $F(t)$ For the Standardized, Doubly-Truncated Normal Distribution

k_L	k_R	Cumulative Probability, $F(t)$																Mean	Std Dev	Cov
		0.01	0.025	0.05	0.1	0.15	0.2	0.5	0.8	0.85	0.9	0.95	0.975	0.99						
-3.0	-2.8	0.0027	0.0067	0.0133	0.0261	0.0385	0.0504	0.1143	0.1682	0.1764	0.1844	0.1923	0.1962	0.1985	0.1096	0.0566	0.5166			
-3.0	-2.6	0.0074	0.0182	0.0354	0.0675	0.0968	0.1237	0.2528	0.3478	0.3616	0.3748	0.3876	0.3939	0.3976	0.2364	0.1113	0.4709			
-3.0	-2.4	0.0151	0.0366	0.0695	0.1273	0.1768	0.2202	0.4082	0.5338	0.5514	0.5682	0.5844	0.5923	0.5969	0.3768	0.1616	0.4288			
-3.0	-2.2	0.0272	0.0643	0.1183	0.2063	0.2768	0.3360	0.5737	0.7230	0.7435	0.7632	0.7819	0.7911	0.7964	0.5272	0.2070	0.3927			
-3.0	-2.0	0.0451	0.1032	0.1825	0.3022	0.3925	0.4655	0.7445	0.9136	0.9367	0.9587	0.9798	0.9900	0.9960	0.6842	0.2480	0.3624			
-3.0	-1.8	0.0702	0.1541	0.2607	0.4107	0.5182	0.6031	0.9173	1.1045	1.1300	1.1544	1.1777	1.1889	1.1956	0.8451	0.2852	0.3375			
-3.0	-1.6	0.1031	0.2162	0.3498	0.5268	0.6492	0.7441	1.0901	1.2951	1.3221	1.3498	1.3754	1.3878	1.3952	1.0077	0.3198	0.3173			
-3.0	-1.4	0.1438	0.2871	0.4457	0.6462	0.7815	0.8854	1.2614	1.4849	1.5156	1.5449	1.5730	1.5866	1.5947	1.1702	0.3526	0.3013			
-3.0	-1.2	0.1913	0.3639	0.5446	0.7654	0.9122	1.0243	1.4300	1.6734	1.7070	1.7392	1.7702	1.7852	1.7941	1.3314	0.3846	0.2889			
-3.0	-1.0	0.2437	0.4432	0.6431	0.8818	1.0391	1.1590	1.5949	1.8602	1.8972	1.9327	1.9670	1.9836	1.9935	1.4899	0.4165	0.2795			
-3.0	-0.8	0.2988	0.5224	0.7389	0.9935	1.1606	1.2880	1.7552	2.0449	2.0857	2.1251	2.1631	2.1817	2.1927	1.6449	0.4488	0.2729			
-3.0	-0.6	0.3545	0.5989	0.8298	1.0988	1.2752	1.4101	1.9097	2.2267	2.2720	2.3159	2.3585	2.3794	2.3918	1.7952	0.4820	0.2685			
-3.0	-0.4	0.4088	0.6710	0.9145	1.1967	1.3820	1.5242	2.0575	2.4050	2.4556	2.5049	2.5529	2.5766	2.5907	1.9400	0.5165	0.2662			
-3.0	-0.2	0.4601	0.7375	0.9920	1.2863	1.4800	1.6293	2.1972	2.5790	2.6357	2.6914	2.7461	2.7731	2.7893	2.0782	0.5523	0.2657			
-3.0	0.0	0.5072	0.7975	1.0615	1.3668	1.5685	1.7246	2.3276	2.7474	2.8114	2.8747	2.9375	2.9688	2.9875	2.2088	0.5894	0.2668			
-3.0	0.2	0.5493	0.8505	1.1228	1.4380	1.6471	1.8095	2.4475	2.9088	2.9814	3.0538	3.1266	3.1632	3.1852	2.3310	0.6278	0.2693			
-3.0	0.4	0.5861	0.8963	1.1757	1.4996	1.7154	1.8838	2.5556	3.0617	3.1442	3.2276	3.3126	3.3560	3.3823	2.4437	0.6671	0.2730			
-3.0	0.6	0.6173	0.9350	1.2204	1.5519	1.7735	1.9473	2.6510	3.2041	3.2978	3.3943	3.4945	3.5465	3.5784	2.5461	0.7068	0.2776			
-3.0	0.8	0.6433	0.9669	1.2573	1.5951	1.8219	2.0002	2.7330	3.3339	3.4403	3.5518	3.6707	3.7338	3.7731	2.6374	0.7464	0.2830			
-3.0	1.0	0.6642	0.9925	1.2869	1.6300	1.8610	2.0433	2.8016	3.4492	3.5691	3.6978	3.8393	3.9167	3.9659	2.7172	0.7849	0.2889			
-3.0	1.2	0.6806	1.0126	1.3102	1.6574	1.8918	2.0772	2.8570	3.5482	3.6820	3.8294	3.9976	4.0932	4.1557	2.7852	0.8216	0.2950			
-3.0	1.4	0.6931	1.0279	1.3278	1.6783	1.9153	2.1033	2.9003	3.6300	3.7775	3.9441	4.1424	4.2607	4.3411	2.8417	0.8556	0.3011			
-3.0	1.6	0.7023	1.0391	1.3409	1.6938	1.9327	2.1225	2.9330	3.6949	3.8546	4.0399	4.2703	4.4160	4.5201	2.8872	0.8861	0.3069			
-3.0	1.8	0.7088	1.0471	1.3501	1.7048	1.9452	2.1364	2.9566	3.7439	3.9141	4.1160	4.3782	4.5548	4.6895	2.9226	0.9125	0.3122			
-3.0	2.0	0.7134	1.0527	1.3565	1.7124	1.9538	2.1459	2.9732	3.7792	3.9576	4.1733	4.4644	4.6732	4.8451	2.9492	0.9344	0.3168			
-3.0	2.2	0.7164	1.0563	1.3608	1.7175	1.9595	2.1523	2.9843	3.8035	3.9878	4.2140	4.5289	4.7681	4.9818	2.9685	0.9520	0.3207			
-3.0	2.4	0.7183	1.0587	1.3635	1.7207	1.9632	2.1564	2.9914	3.8194	4.0078	4.2413	4.5742	4.8390	5.0946	2.9819	0.9654	0.3238			
-3.0	2.6	0.7195	1.0602	1.3652	1.7227	1.9655	2.1589	2.9958	3.8293	4.0204	4.2588	4.6040	4.8880	5.1807	2.9908	0.9752	0.3261			
-3.0	2.8	0.7202	1.0610	1.3662	1.7239	1.9668	2.1604	2.9985	3.8353	4.0280	4.2693	4.6224	4.9195	5.2409	2.9965	0.9820	0.3277			
-3.0	3.0	0.7206	1.0615	1.3668	1.7246	1.9676	2.1613	3.0000	3.8387	4.0324	4.2754	4.6332	4.9385	5.2794	3.0000	0.9866	0.3289			

Table 1. (continued)

k_L	k_R	0.01	0.025	0.05	0.1	0.15	0.2	0.5	0.8	0.85	0.9	0.95	0.975	0.99	Mean	Std Dev	Cov
-2.8	-2.6	0.0026	0.0066	0.0131	0.0257	0.0378	0.0496	0.1133	0.1677	0.1760	0.1842	0.1922	0.1961	0.1984	0.1089	0.0564	0.5175
-2.8	-2.4	0.0071	0.0174	0.0340	0.0650	0.0937	0.1202	0.2494	0.3463	0.3604	0.3740	0.3872	0.3936	0.3975	0.2339	0.1118	0.4779
-2.8	-2.2	0.0141	0.0342	0.0654	0.1208	0.1691	0.2119	0.4014	0.5310	0.5493	0.5668	0.5837	0.5919	0.5968	0.3715	0.1630	0.4389
-2.8	-2.0	0.0246	0.0587	0.1093	0.1936	0.2626	0.3214	0.5633	0.7189	0.7404	0.7611	0.7809	0.7906	0.7962	0.5185	0.2099	0.4050
-2.8	-1.8	0.0399	0.0925	0.1664	0.2815	0.3706	0.4437	0.7304	0.9081	0.9326	0.9560	0.9784	0.9893	0.9958	0.6716	0.2528	0.3765
-2.8	-1.6	0.0606	0.1360	0.2356	0.3809	0.4881	0.5739	0.8994	1.0977	1.1250	1.1510	1.1760	1.1881	1.1953	0.8284	0.2924	0.3530
-2.8	-1.4	0.0873	0.1887	0.3143	0.4875	0.6106	0.7075	1.0682	1.2868	1.3169	1.3457	1.3734	1.3868	1.3948	0.9866	0.3295	0.3340
-2.8	-1.2	0.1199	0.2488	0.3993	0.5973	0.7344	0.8410	1.2352	1.4749	1.5081	1.5399	1.5705	1.5854	1.1445	0.3651	0.3190	0.3190
-2.8	-1.0	0.1576	0.3139	0.4870	0.7068	0.8564	0.9720	1.3990	1.6614	1.6980	1.7333	1.7672	1.7837	1.7935	1.3006	0.3999	0.3075
-2.8	-0.8	0.1990	0.3813	0.5744	0.8134	0.9743	1.0984	1.5585	1.8458	1.8864	1.9255	1.9633	1.9818	1.9928	1.4537	0.4347	0.2990
-2.8	-0.6	0.2426	0.4485	0.6591	0.9151	1.0865	1.2186	1.7125	2.0275	2.0726	2.1163	2.1587	2.1795	2.1918	1.6027	0.4698	0.2931
-2.8	-0.4	0.2864	0.5133	0.7391	1.0104	1.1915	1.3313	1.8598	2.2057	2.2562	2.3052	2.3531	2.3767	2.3907	1.7464	0.5058	0.2896
-2.8	-0.2	0.3290	0.5741	0.8131	1.0980	1.2881	1.4354	1.9993	2.3796	2.4362	2.4917	2.5462	2.5732	2.5893	1.8838	0.5428	0.2881
-2.8	0.0	0.3691	0.6297	0.8800	1.1770	1.3756	1.5300	2.1295	2.5480	2.6119	2.6750	2.7376	2.7688	2.7875	2.0139	0.5810	0.2885
-2.8	0.2	0.4055	0.6793	0.9393	1.2471	1.4534	1.6144	2.2493	2.7094	2.7818	2.8541	2.9267	2.9633	2.9853	2.1357	0.6202	0.2904
-2.8	0.4	0.4378	0.7225	0.9907	1.3079	1.5212	1.6883	2.3573	2.8623	2.9446	3.0279	3.1128	3.1561	3.1823	2.2480	0.6601	0.2936
-2.8	0.6	0.4656	0.7592	1.0342	1.3596	1.5790	1.7515	2.4526	3.0047	3.0983	3.1946	3.2947	3.3466	3.3784	2.3502	0.7004	0.2980
-2.8	0.8	0.4888	0.7896	1.0702	1.4024	1.6270	1.8042	2.5346	3.1346	3.2408	3.3522	3.4709	3.5339	3.5732	2.4413	0.7404	0.3033
-2.8	1.0	0.5077	0.8142	1.0992	1.4370	1.6659	1.8471	2.6031	3.2498	3.3696	3.4982	3.6395	3.7168	3.7659	2.5210	0.7793	0.3091
-2.8	1.2	0.5225	0.8334	1.1220	1.4641	1.6965	1.8809	2.6585	3.3489	3.4826	3.6299	3.7978	3.8933	3.9557	2.5889	0.8162	0.3153
-2.8	1.4	0.5339	0.8481	1.1393	1.4848	1.7199	1.9069	2.7018	3.4308	3.5781	3.7446	3.9427	4.0609	4.1412	2.6453	0.8504	0.3215
-2.8	1.6	0.5423	0.8589	1.1521	1.5001	1.7373	1.9261	2.7345	3.4956	3.6553	3.8405	4.0706	4.2162	4.3202	2.6907	0.8811	0.3274
-2.8	1.8	0.5483	0.8666	1.1612	1.5110	1.7497	1.9399	2.7582	3.5447	3.7148	3.9166	4.1786	4.3550	4.4897	2.7261	0.9076	0.3329
-2.8	2.0	0.5524	0.8719	1.1675	1.5186	1.7582	1.9494	2.7747	3.5800	3.7583	3.9739	4.2648	4.4735	4.6453	2.7527	0.9296	0.3377
-2.8	2.2	0.5552	0.8755	1.1717	1.5236	1.7639	1.9557	2.7858	3.6043	3.7886	4.0146	4.3294	4.5685	4.7820	2.7720	0.9473	0.3417
-2.8	2.4	0.5570	0.8777	1.1744	1.5268	1.7676	1.9598	2.7929	3.6202	3.8086	4.0420	4.3747	4.6394	4.8949	2.7854	0.9608	0.3449
-2.8	2.6	0.5581	0.8791	1.1761	1.5288	1.7699	1.9623	2.7974	3.6302	3.8212	4.0594	4.4045	4.6885	4.9811	2.7943	0.9706	0.3474
-2.8	2.8	0.5587	0.8800	1.1770	1.5300	1.7712	1.9638	2.8000	3.6362	3.8288	4.0700	4.4230	4.7200	5.0413	2.8000	0.9775	0.3491
-2.8	3.0	0.5591	0.8805	1.1776	1.5307	1.7720	1.9647	2.8015	3.6396	3.8332	4.0761	4.4338	4.7390	5.0798	2.8035	0.9820	0.3503

Table 1. (continued)

k_L	k_R	Cumulative Probability, $F(t)$																Mean	Std Dev	CoV
		0.01	0.025	0.05	0.1	0.15	0.2	0.5	0.8	0.85	0.9	0.95	0.975	0.99						
-2.6	-2.4	0.0026	0.0065	0.0128	0.0252	0.0372	0.0489	0.1123	0.1672	0.1756	0.1839	0.1920	0.1960	0.1984	0.1083	0.0566	0.5229			
-2.6	-2.2	0.0067	0.0166	0.0326	0.0627	0.0906	0.1168	0.2458	0.3446	0.3591	0.3732	0.3868	0.3934	0.3974	0.2313	0.1123	0.4856			
-2.6	-2.0	0.0131	0.0319	0.0615	0.1146	0.1615	0.2036	0.3944	0.5280	0.5470	0.5653	0.5830	0.5915	0.5966	0.3661	0.1644	0.4491			
-2.6	-1.8	0.0224	0.0537	0.1009	0.1813	0.2486	0.3067	0.5523	0.7144	0.7371	0.7589	0.7798	0.7900	0.7960	0.5095	0.2127	0.4175			
-2.6	-1.6	0.0353	0.0828	0.1513	0.2613	0.3487	0.4216	0.7152	0.9022	0.9282	0.9531	0.9770	0.9886	0.9955	0.6586	0.2575	0.3910			
-2.6	-1.4	0.0523	0.1197	0.2118	0.3515	0.4575	0.5438	0.8799	1.0901	1.1193	1.1473	1.1741	1.1872	1.1949	0.8109	0.2993	0.3692			
-2.6	-1.2	0.0738	0.1638	0.2804	0.4481	0.5709	0.6690	1.0441	1.2774	1.3099	1.3411	1.3711	1.3857	1.3943	0.9642	0.3390	0.3516			
-2.6	-1.0	0.0995	0.2138	0.3543	0.5475	0.6852	0.7940	1.2060	1.4634	1.4995	1.5342	1.5677	1.5840	1.5936	1.1169	0.3774	0.3379			
-2.6	-0.8	0.1288	0.2677	0.4305	0.6465	0.7976	0.9161	1.3642	1.6475	1.6876	1.7263	1.7637	1.7820	1.7928	1.2674	0.4150	0.3275			
-2.6	-0.6	0.1607	0.3233	0.5062	0.7425	0.9057	1.0332	1.5172	1.8289	1.8737	1.9170	1.9590	1.9796	1.9919	1.4143	0.4526	0.3200			
-2.6	-0.4	0.1938	0.3785	0.5793	0.8335	1.0076	1.1436	1.6639	2.0070	2.0571	2.1058	2.1534	2.1768	2.1908	1.5565	0.4906	0.3152			
-2.6	-0.2	0.2270	0.4316	0.6479	0.9178	1.1021	1.2461	1.8029	2.1808	2.2371	2.2922	2.3465	2.3733	2.3894	1.6928	0.5293	0.3126			
-2.6	0.0	0.2589	0.4810	0.7107	0.9945	1.1879	1.3394	1.9328	2.3491	2.4127	2.4755	2.5379	2.5690	2.5876	1.8220	0.5687	0.3121			
-2.6	0.2	0.2887	0.5257	0.7669	1.0628	1.2645	1.4229	2.0523	2.5105	2.5826	2.6547	2.7270	2.7634	2.7853	1.9431	0.6090	0.3134			
-2.6	0.4	0.3154	0.5651	0.8158	1.1222	1.3313	1.4961	2.1602	2.6634	2.7454	2.8284	2.9131	2.9562	2.9824	2.0550	0.6499	0.3162			
-2.6	0.6	0.3388	0.5989	0.8576	1.1729	1.3884	1.5587	2.2554	2.8058	2.8991	2.9952	3.0950	3.1467	3.1785	2.1567	0.6908	0.3203			
-2.6	0.8	0.3585	0.6270	0.8922	1.2149	1.4359	1.6111	2.3373	2.9357	3.0416	3.1528	3.2713	3.3341	3.3732	2.2476	0.7314	0.3254			
-2.6	1.0	0.3746	0.6498	0.9202	1.2489	1.4744	1.6537	2.4058	3.0510	3.1705	3.2988	3.4399	3.5170	3.5660	2.3270	0.7708	0.3312			
-2.6	1.2	0.3874	0.6678	0.9422	1.2756	1.5047	1.6874	2.4612	3.1501	3.2836	3.4306	3.5982	3.6935	3.7558	2.3948	0.8081	0.3374			
-2.6	1.4	0.3973	0.6815	0.9589	1.2960	1.5279	1.7131	2.5045	3.2321	3.3792	3.5454	3.7432	3.8612	3.9414	2.4511	0.8426	0.3438			
-2.6	1.6	0.4046	0.6917	0.9713	1.3111	1.5451	1.7323	2.5371	3.2970	3.4564	3.6414	3.8712	4.0165	4.1204	2.4965	0.8735	0.3499			
-2.6	1.8	0.4098	0.6989	0.9801	1.3219	1.5574	1.7460	2.5608	3.3461	3.5160	3.7176	3.9793	4.1555	4.2899	2.5319	0.9002	0.3556			
-2.6	2.0	0.4134	0.7039	0.9863	1.3293	1.5659	1.7554	2.5773	3.3815	3.5596	3.7749	4.0656	4.2740	4.4456	2.5585	0.9225	0.3606			
-2.6	2.2	0.4159	0.7073	0.9903	1.3343	1.5716	1.7618	2.5884	3.4058	3.5899	3.8157	4.1303	4.3691	4.5824	2.5777	0.9402	0.3647			
-2.6	2.4	0.4174	0.7094	0.9929	1.3375	1.5752	1.7658	2.5956	3.4217	3.6099	3.8431	4.1756	4.4401	4.6954	2.5911	0.9538	0.3681			
-2.6	2.6	0.4184	0.7107	0.9945	1.3394	1.5775	1.7683	2.6000	3.4317	3.6225	3.8606	4.2055	4.4893	4.7816	2.6000	0.9637	0.3707			
-2.6	2.8	0.4189	0.7115	0.9955	1.3406	1.5788	1.7698	2.6026	3.4377	3.6301	3.8712	4.2239	4.5209	4.8419	2.6057	0.9706	0.3725			
-2.6	3.0	0.4193	0.7120	0.9960	1.3412	1.5796	1.7707	2.6042	3.4411	3.6345	3.8773	4.2348	4.5398	4.8805	2.6092	0.9752	0.3738			

Table 1. (continued)

k_L	k_R	Cumulative Probability, $F(t)$																Mean	Std Dev	CoV
		0.01	0.025	0.05	0.1	0.15	0.2	0.5	0.8	0.85	0.9	0.95	0.975	0.99						
-2.4	-2.2	0.0025	0.0063	0.0126	0.0247	0.0366	0.0481	0.1114	0.1666	0.1752	0.1836	0.1919	0.1960	0.1984	0.1076	0.0570	0.5296			
-2.4	-2.0	0.0065	0.0159	0.0313	0.0604	0.0877	0.1133	0.2423	0.3429	0.3578	0.3723	0.3863	0.3932	0.3973	0.2288	0.1129	0.4934			
-2.4	-1.8	0.0122	0.0299	0.0578	0.1086	0.1542	0.1955	0.3871	0.5448	0.5446	0.5637	0.5822	0.5911	0.5965	0.3607	0.1657	0.4595			
-2.4	-1.6	0.0203	0.0490	0.0930	0.1695	0.2348	0.2921	0.5408	0.7096	0.7335	0.7565	0.7786	0.7894	0.7958	0.5004	0.2153	0.4303			
-2.4	-1.4	0.0312	0.0741	0.1372	0.2417	0.3269	0.3992	0.6991	0.8956	0.9233	0.9498	0.9753	0.9878	0.9951	0.2619	0.14059	0.4059			
-2.4	-1.2	0.0452	0.1051	0.1897	0.3227	0.4267	0.5129	0.8588	1.0817	1.1130	1.1430	1.1720	1.1861	1.1945	0.7926	0.3059	0.3860			
-2.4	-1.0	0.0623	0.1416	0.2486	0.4091	0.5304	0.6291	1.0176	1.2668	1.3019	1.3358	1.3684	1.3843	1.3938	0.9406	0.3481	0.3701			
-2.4	-0.8	0.0824	0.1823	0.3116	0.4977	0.6346	0.7447	1.1736	1.4503	1.4896	1.5276	1.5643	1.5823	1.5930	1.0875	0.3892	0.3578			
-2.4	-0.6	0.1047	0.2259	0.3763	0.5855	0.7365	0.8569	1.3252	1.6313	1.6754	1.7181	1.7596	1.7799	1.7920	1.2317	0.4296	0.3488			
-2.4	-0.4	0.1286	0.2705	0.4403	0.6702	0.8338	0.9638	1.4708	1.8091	1.8586	1.9068	1.9539	1.9771	1.9908	1.3718	0.4700	0.3426			
-2.4	-0.2	0.1531	0.3145	0.5016	0.7498	0.9248	1.0636	1.6090	1.9827	2.0385	2.0932	2.1469	2.1736	2.1894	1.5064	0.5107	0.3390			
-2.4	0.0	0.1772	0.3563	0.5587	0.8228	1.0080	1.1550	1.7384	2.1509	2.2140	2.2764	2.3383	2.3692	2.3877	1.6344	0.5518	0.3376			
-2.4	0.2	0.2001	0.3949	0.6103	0.8883	1.0826	1.2370	1.8575	2.3123	2.3840	2.4556	2.5274	2.5636	2.5854	1.7545	0.5935	0.3383			
-2.4	0.4	0.2211	0.4293	0.6558	0.9456	1.1480	1.3090	1.9651	2.4652	2.5468	2.6293	2.7135	2.7564	2.7825	1.8656	0.6354	0.3406			
-2.4	0.6	0.2397	0.4591	0.6948	0.9946	1.2039	1.3709	2.0601	2.6076	2.7005	2.7961	2.8955	2.9470	2.9786	1.9668	0.6774	0.3444			
-2.4	0.8	0.2555	0.4842	0.7273	1.0354	1.2506	1.4226	2.1419	2.7376	2.8431	2.9538	3.0718	3.1344	3.1734	2.0573	0.7187	0.3493			
-2.4	1.0	0.2686	0.5047	0.7537	1.0685	1.2885	1.4647	2.2103	2.8530	2.9721	3.1000	3.2405	3.3173	3.3661	2.1364	0.7587	0.3551			
-2.4	1.2	0.2791	0.5209	0.7746	1.0946	1.3184	1.4981	2.2657	2.9522	3.0853	3.2319	3.3990	3.4939	3.5560	2.2041	0.7966	0.3614			
-2.4	1.4	0.2872	0.5333	0.7905	1.1145	1.3412	1.5236	2.3089	3.0342	3.1810	3.3468	3.5440	3.6617	3.7416	2.2602	0.8315	0.3679			
-2.4	1.6	0.2932	0.5425	0.8022	1.1292	1.3582	1.5426	2.3416	3.0992	3.2584	3.4429	3.6722	3.8171	3.9207	2.3055	0.8627	0.3742			
-2.4	1.8	0.2975	0.5491	0.8107	1.1397	1.3703	1.5561	2.3652	3.1484	3.3180	3.5192	3.7805	3.9562	4.0903	2.3408	0.8897	0.3801			
-2.4	2.0	0.3005	0.5537	0.8165	1.1470	1.3787	1.5655	2.3818	3.1839	3.3617	3.5767	3.8669	4.0749	4.2461	2.3674	0.9122	0.3853			
-2.4	2.2	0.3025	0.5567	0.8203	1.1518	1.3843	1.5718	2.3929	3.2082	3.3920	3.6176	3.9317	4.1701	4.3830	2.3866	0.9301	0.3897			
-2.4	2.4	0.3038	0.5587	0.8228	1.1550	1.3879	1.5758	2.4000	3.2242	3.4121	3.6450	3.9772	4.2413	4.4962	2.4000	0.9438	0.3932			
-2.4	2.6	0.3046	0.5599	0.8244	1.1569	1.3901	1.5783	2.4044	3.2342	3.4248	3.6626	4.0071	4.2906	4.5826	2.4089	0.9538	0.3959			
-2.4	2.8	0.3051	0.5606	0.8253	1.1580	1.3914	1.5798	2.4071	3.2402	3.4324	3.6732	4.0256	4.3223	4.6430	2.4146	0.9608	0.3979			
-2.4	3.0	0.3054	0.5610	0.8258	1.1587	1.3922	1.5806	2.4086	3.2436	3.4368	3.6793	4.0364	4.3413	4.6817	2.4181	0.9654	0.3992			

Table 1. (continued)

k_c	k_e	Cumulative Probability, $F(t)$															Mean	Std Dev	CoV
		0.01	0.025	0.05	0.1	0.15	0.2	0.5	0.8	0.85	0.9	0.95	0.975	0.99					
-2.2	-2.0	0.0025	0.0062	0.0123	0.0243	0.0360	0.0473	0.1104	0.1661	0.1748	0.1833	0.1917	0.1959	0.1984	0.1070	0.0573	0.5360		
-2.2	-1.8	0.0062	0.0153	0.0300	0.0582	0.0848	0.1100	0.2386	0.3412	0.3565	0.3714	0.3859	0.3930	0.3972	0.2262	0.1134	0.5011		
-2.2	-1.6	0.0114	0.0280	0.0543	0.1029	0.1470	0.1875	0.3796	0.5215	0.5421	0.5620	0.5813	0.5907	0.5963	0.3552	0.1669	0.4699		
-2.2	-1.4	0.0185	0.0448	0.0857	0.1583	0.2214	0.2775	0.5287	0.7045	0.7296	0.7539	0.7773	0.7887	0.7955	0.4910	0.2177	0.4434		
-2.2	-1.2	0.0277	0.0663	0.1243	0.2230	0.3055	0.3768	0.6818	0.8885	0.9179	0.9462	0.9735	0.9869	0.9948	0.6312	0.2660	0.4214		
-2.2	-1.0	0.0391	0.0922	0.1693	0.2948	0.3960	0.4815	0.8360	1.0722	1.1058	1.1383	1.1696	1.1849	1.1940	0.7734	0.3121	0.4035		
-2.2	-0.8	0.0527	0.1219	0.2191	0.3710	0.4896	0.5881	0.9886	1.2547	1.2928	1.3297	1.3653	1.3828	1.3931	0.9158	0.3567	0.3895		
-2.2	-0.6	0.0681	0.1547	0.2719	0.4486	0.5832	0.6935	1.1378	1.4352	1.4782	1.5199	1.5605	1.5804	1.5922	1.0563	0.4003	0.3789		
-2.2	-0.4	0.0849	0.1891	0.3256	0.5250	0.6740	0.7952	1.2817	1.6125	1.6611	1.7084	1.7547	1.7774	1.7910	1.1936	0.4434	0.3715		
-2.2	-0.2	0.1024	0.2239	0.3782	0.5981	0.7600	0.8910	1.4187	1.7858	1.8408	1.8947	1.9477	1.9739	1.9896	1.3260	0.4864	0.3668		
-2.2	0.0	0.1200	0.2578	0.4281	0.6660	0.8394	0.9794	1.5472	1.9538	2.0162	2.0779	2.1390	2.1695	2.1878	1.4523	0.5295	0.3646		
-2.2	0.2	0.1370	0.2895	0.4740	0.7275	0.9110	1.0592	1.6657	2.1151	2.1861	2.2570	2.3282	2.3640	2.3856	1.5711	0.5728	0.3646		
-2.2	0.4	0.1528	0.3183	0.5148	0.7817	0.9742	1.1295	1.7729	2.2680	2.3489	2.4308	2.5143	2.5568	2.5826	1.6812	0.6161	0.3665		
-2.2	0.6	0.1669	0.3436	0.5502	0.8284	1.0284	1.1901	1.8677	2.4105	2.5028	2.5977	2.6963	2.7474	2.7788	1.7817	0.6592	0.3700		
-2.2	0.8	0.1791	0.3651	0.5800	0.8674	1.0738	1.2409	1.9493	2.5406	2.6455	2.7555	2.8727	2.9349	2.9735	1.8717	0.7015	0.3748		
-2.2	1.0	0.1893	0.3827	0.6043	0.8991	1.1107	1.2823	2.0176	2.6561	2.7746	2.9018	3.0415	3.1179	3.1664	1.9504	0.7423	0.3806		
-2.2	1.2	0.1975	0.3968	0.6235	0.9242	1.1399	1.3151	2.0729	2.7555	2.8880	3.0339	3.2001	3.2946	3.3563	2.0178	0.7808	0.3870		
-2.2	1.4	0.2038	0.4076	0.6382	0.9433	1.1623	1.3403	2.1161	2.8377	2.9839	3.1491	3.3454	3.4625	3.5419	2.0738	0.8163	0.3936		
-2.2	1.6	0.2086	0.4157	0.6491	0.9575	1.1789	1.3590	2.1487	2.9029	3.0615	3.2454	3.4738	3.6181	3.7211	2.1190	0.8480	0.4002		
-2.2	1.8	0.2120	0.4215	0.6570	0.9677	1.1908	1.3724	2.1724	2.9522	3.1213	3.3219	3.5823	3.7574	3.8909	2.1542	0.8753	0.4063		
-2.2	2.0	0.2144	0.4255	0.6624	0.9747	1.1990	1.3816	2.1889	2.9878	3.1651	3.3796	3.6690	3.8764	4.0469	2.1808	0.8981	0.4118		
-2.2	2.2	0.2160	0.4281	0.6660	0.9794	1.2044	1.3878	2.2000	3.0122	3.1956	3.4206	3.7340	3.9719	4.1840	2.2000	0.9162	0.4165		
-2.2	2.4	0.2170	0.4298	0.6683	0.9824	1.2080	1.3918	2.2071	3.0282	3.2157	3.4482	3.7797	4.0433	4.2975	2.2134	0.9301	0.4202		
-2.2	2.6	0.2176	0.4309	0.6697	0.9843	1.2101	1.3942	2.2116	3.0382	3.2284	3.4657	3.8097	4.0927	4.3842	2.2223	0.9402	0.4231		
-2.2	2.8	0.2180	0.4315	0.6706	0.9854	1.2114	1.3957	2.2142	3.0443	3.2361	3.4764	3.8283	4.1245	4.4448	2.2280	0.9473	0.4252		
-2.2	3.0	0.2182	0.4319	0.6711	0.9860	1.2122	1.3965	2.2157	3.0477	3.2405	3.4826	3.8392	4.1437	4.4836	2.2315	0.9520	0.4266		

Table 1. (continued)

k_L	k_R	0.01	0.025	0.05	0.1	0.15	0.2	0.5	0.8	0.85	0.9	0.95	0.975	0.99	Mean	Std Dev	Cov
-2.0	-1.8	0.0024	0.0061	0.0121	0.0238	0.0353	0.0466	0.1094	0.1656	0.1744	0.1830	0.1916	0.1958	0.1983	0.1063	0.0575	0.5412
-2.0	-1.6	0.0059	0.0146	0.0288	0.0561	0.0820	0.1067	0.2349	0.3393	0.3551	0.3704	0.3854	0.3927	0.3971	0.2237	0.1137	0.5085
-2.0	-1.4	0.0106	0.0262	0.0510	0.0975	0.1401	0.1796	0.3719	0.5179	0.5393	0.5602	0.5804	0.5902	0.5961	0.3496	0.1679	0.4804
-2.0	-1.2	0.0168	0.0410	0.0790	0.1476	0.2084	0.2632	0.6098	0.7254	0.7510	0.7759	0.7880	0.7952	0.4814	0.2199	0.4567	0.4567
-2.0	-1.0	0.0246	0.0593	0.1125	0.2052	0.2846	0.3545	0.6366	0.8005	0.9119	0.9432	0.9715	0.9859	0.9944	0.6168	0.2697	0.4373
-2.0	-0.8	0.0339	0.0808	0.1507	0.2683	0.3659	0.4500	0.8114	1.0617	1.0979	1.1329	1.1659	1.1836	1.1935	0.7536	0.3178	0.4217
-2.0	-0.6	0.0446	0.1049	0.1923	0.3344	0.4492	0.5466	0.9571	1.2411	1.2825	1.3227	1.3618	1.3810	1.3924	0.8897	0.3646	0.4098
-2.0	-0.4	0.0563	0.1308	0.2357	0.4012	0.5318	0.6413	1.0985	1.4178	1.4649	1.5109	1.5559	1.5781	1.5912	1.0235	0.4106	0.4012
-2.0	-0.2	0.0688	0.1575	0.2793	0.4663	0.6112	0.7317	1.2337	1.5907	1.6443	1.6970	1.7488	1.7745	1.7898	1.1531	0.4560	0.3955
-2.0	0.0	0.0815	0.1839	0.3214	0.5277	0.6855	0.8160	1.3609	1.7584	1.8196	1.8801	1.9401	1.9701	1.9880	1.2772	0.5013	0.3925
-2.0	0.2	0.0939	0.2092	0.3607	0.5841	0.7532	0.8926	1.4785	1.9196	1.9894	2.0592	2.1293	2.1645	2.1858	1.3943	0.5465	0.3919
-2.0	0.4	0.1055	0.2324	0.3962	0.6343	0.8132	0.9605	1.5850	2.0725	2.1523	2.2331	2.3154	2.3574	2.3829	1.5033	0.5914	0.3934
-2.0	0.6	0.1160	0.2531	0.4273	0.6778	0.8651	1.0192	1.6794	2.2151	2.3063	2.4001	2.4975	2.5481	2.5790	1.6028	0.6358	0.3967
-2.0	0.8	0.1251	0.2708	0.4537	0.7144	0.9087	1.0686	1.7607	2.3453	2.4491	2.5581	2.6741	2.7356	2.7739	1.6921	0.6792	0.4014
-2.0	1.0	0.1328	0.2855	0.4754	0.7443	0.9442	1.1090	1.8288	2.4611	2.5786	2.7046	2.8431	2.9187	2.9667	1.7704	0.7209	0.4072
-2.0	1.2	0.1390	0.2972	0.4926	0.7680	0.9724	1.1411	1.8840	2.5607	2.6922	2.8370	3.0020	3.0956	3.1567	1.8374	0.7603	0.4138
-2.0	1.4	0.1438	0.3063	0.5059	0.7861	0.9941	1.1657	1.9272	2.6432	2.7884	2.9526	3.1475	3.2637	3.3425	1.8932	0.7964	0.4207
-2.0	1.6	0.1475	0.3131	0.5158	0.7996	1.0101	1.1840	1.9598	2.7086	2.8663	3.0492	3.2763	3.4196	3.5218	1.9383	0.8287	0.4275
-2.0	1.8	0.1501	0.3180	0.5228	0.8093	1.0216	1.1971	1.9835	2.7581	2.9264	3.1261	3.3852	3.5593	3.6918	1.9735	0.8565	0.4340
-2.0	2.0	0.1519	0.3214	0.5277	0.8160	1.0296	1.2062	2.0000	2.7938	2.9704	3.1840	3.4723	3.6786	3.8481	2.0000	0.8796	0.4398
-2.0	2.2	0.1531	0.3236	0.5310	0.8204	1.0349	1.2122	2.0111	2.8184	3.0010	3.2253	3.5376	3.7745	3.9856	2.0192	0.8981	0.4448
-2.0	2.4	0.1539	0.3251	0.5331	0.8233	1.0383	1.2161	2.0182	2.8345	3.0213	3.2530	3.5835	3.8463	4.0995	2.0326	0.9122	0.4488
-2.0	2.6	0.1544	0.3260	0.5344	0.8251	1.0404	1.2185	2.0227	2.8446	3.0341	3.2707	3.6138	3.8961	4.1866	2.0415	0.9225	0.4518
-2.0	2.8	0.1547	0.3265	0.5352	0.8261	1.0417	1.2200	2.0253	2.8506	3.0418	3.2814	3.6325	3.9281	4.2476	2.0473	0.9296	0.4541
-2.0	3.0	0.1549	0.3268	0.5356	0.8267	1.0424	1.2208	2.0268	2.8543	3.0462	3.2844	3.6434	3.9473	4.2866	2.0508	0.9344	0.4556
-1.8	-1.6	0.0024	0.0059	0.0118	0.0234	0.0347	0.0459	0.1084	0.1650	0.1739	0.1828	0.1914	0.1957	0.1983	0.1057	0.0576	0.5454
-1.8	-1.4	0.0057	0.0140	0.0277	0.0541	0.0793	0.1035	0.2312	0.3374	0.3536	0.3694	0.3849	0.3925	0.3970	0.2211	0.1140	0.5158
-1.8	-1.2	0.0099	0.0245	0.0480	0.0923	0.1334	0.1720	0.3640	0.5141	0.5365	0.5582	0.5794	0.5897	0.5959	0.3439	0.1688	0.4910
-1.8	-1.0	0.0153	0.0376	0.0728	0.1375	0.1958	0.2492	0.5029	0.6928	0.7208	0.7479	0.7743	0.7872	0.7949	0.4717	0.2218	0.4703
-1.8	-0.8	0.0218	0.0531	0.1017	0.1884	0.2644	0.3325	0.6443	0.8719	0.9053	0.9377	0.9693	0.9847	0.9939	0.6021	0.2731	0.4536
-1.8	-0.6	0.0294	0.0708	0.1339	0.2432	0.3366	0.4187	0.7852	1.0499	1.0889	1.1268	1.1638	1.1820	1.1928	0.7331	0.3229	0.4405
-1.8	-0.4	0.0378	0.0901	0.1682	0.2999	0.4098	0.5050	0.9230	1.2256	1.2707	1.3146	1.3577	1.3790	1.3916	0.8626	0.3717	0.4309
-1.8	-0.2	0.0467	0.1104	0.2033	0.3563	0.4814	0.5887	1.0557	1.3978	1.4496	1.5004	1.5505	1.5753	1.5901	0.9890	0.4197	0.4244
-1.8	0.0	0.0559	0.1307	0.2379	0.4105	0.5494	0.6677	1.1810	1.5652	1.6246	1.6834	1.7418	1.7709	1.7884	1.1105	0.4672	0.4208
-1.8	0.2	0.0649	0.1504	0.2707	0.4609	0.6121	0.7401	1.2973	1.7262	1.7944	1.8625	1.9310	1.9654	1.9861	1.2256	0.5144	0.4197
-1.8	0.4	0.0735	0.1687	0.3008	0.5063	0.6682	0.8048	1.4030	1.8791	1.9573	2.0365	2.1172	2.1583	2.1832	1.3330	0.5610	0.4208
-1.8	0.6	0.0812	0.1852	0.3273	0.5459	0.7170	0.8611	1.4967	2.0218	2.1115	2.2036	2.2994	2.3490	2.3794	1.4314	0.6069	0.4240
-1.8	0.8	0.0880	0.1994	0.3501	0.5796	0.7582	0.9087	1.5777	2.1523	2.2546	2.3619	2.4762	2.5367	2.5743	1.5198	0.6515	0.4287
-1.8	1.0	0.0938	0.2113	0.3689	0.6072	0.7921	0.9477	1.6456	2.2684	2.3844	2.5089	2.6455	2.7200	2.7673	1.5976	0.6944	0.4346
-1.8	1.2	0.0985	0.2209	0.3840	0.6292	0.8189	0.9787	1.7007	2.3684	2.4985	2.6417	2.8047	2.8971	2.9574	1.6643	0.7347	0.4414
-1.8	1.4	0.1021	0.2283	0.3956	0.6461	0.8396	1.0025	1.7438	2.4513	2.5952	2.7578	2.9507	3.0655	3.1433	1.7199	0.7716	0.4486
-1.8	1.6	0.1048	0.2339	0.4042	0.6586	0.8550	1.0203	1.7763	2.5171	2.6735	2.8549	3.0801	3.2219	3.3229	1.7648	0.8045	0.4559
-1.8	1.8	0.1068	0.2379	0.4105	0.6677	0.8660	1.0331	1.8000	2.5669	2.7340	2.9323	3.1895	3.3621	3.4932	1.8000	0.8329	0.4627
-1.8	2.0	0.1082	0.2407	0.4148	0.6739	0.8736	1.0419	1.8165	2.6029	2.7784	2.9907	3.2772	3.4820	3.6499	1.8265	0.8565	0.4689
-1.8	2.2	0.1091	0.2426	0.4177	0.6781	0.8787	1.0478	1.8276	2.6276	2.8092	3.0323	3.3430	3.5785	3.7880	1.8458	0.8753	0.4742
-1.8	2.4	0.1097	0.2438	0.4195	0.6808	0.8820	1.0516	1.8348	2.6439	2.8297	3.0603	3.3893	3.6509	3.9025	1.8592	0.8897	0.4786
-1.8	2.6	0.1101	0.2445	0.4207	0.6824	0.8840	1.0539	1.8392	2.6540	2.8426	3.0781	3.4199	3.7011	3.9902	1.8681	0.9002	0.4819
-1.8	2.8	0.1103	0.2450	0.4214	0.6834	0.8852	1.0553	1.8418	2.6601	2.8503	3.0890	3.4388	3.7334	4.0517	1.8739	0.9076	0.4843
-1.8	3.0	0.1105	0.2452	0.4218	0.6840	0.8859	1.0561	1.8434	2.6636	2.8548	3.0952	3.4499	3.7529	4.0912	1.8774	0.9125	0.4860

Table 1. (continued)

K_d	K_B	Cumulative Probability, $F(t)$																Mean	Std Dev	Cov
		0.01	0.025	0.05	0.1	0.15	0.2	0.5	0.8	0.85	0.9	0.95	0.975	0.99						
-1.6	-1.4	0.0023	0.0058	0.0116	0.0230	0.0342	0.0452	0.1075	0.1644	0.1735	0.1824	0.1913	0.1957	0.1983	0.1050	0.0576	0.5489			
-1.6	-1.2	0.0054	0.0134	0.0266	0.0521	0.0767	0.1004	0.2274	0.3354	0.3521	0.3684	0.3843	0.3922	0.3969	0.2185	0.1143	0.5231			
-1.6	-1.0	0.0093	0.0230	0.0452	0.0874	0.1270	0.1645	0.3559	0.5101	0.5334	0.5561	0.5783	0.5892	0.5957	0.3382	0.1696	0.5017			
-1.6	-0.8	0.0140	0.0344	0.0671	0.1280	0.1838	0.2355	0.4892	0.6863	0.7158	0.7445	0.7726	0.7864	0.7946	0.4617	0.2235	0.4841			
-1.6	-0.6	0.0195	0.0476	0.0920	0.1727	0.2451	0.3110	0.6240	0.8623	0.8980	0.9328	0.9667	0.9835	0.9934	0.5870	0.2761	0.4703			
-1.6	-0.4	0.0256	0.0622	0.1189	0.2199	0.3085	0.3880	0.7573	1.0367	1.0788	1.1200	1.1603	1.1803	1.1921	0.7119	0.3274	0.4599			
-1.6	-0.2	0.0322	0.0775	0.1469	0.2678	0.3718	0.4640	0.8865	1.2081	1.2571	1.3054	1.3530	1.3765	1.3906	0.8345	0.3779	0.4528			
-1.6	0.0	0.0389	0.0932	0.1749	0.3146	0.4329	0.5368	1.0094	1.3749	1.4318	1.4882	1.5442	1.5721	1.5888	0.9531	0.4276	0.4486			
-1.6	0.2	0.0456	0.1085	0.2018	0.3587	0.4899	0.6043	1.1240	1.5357	1.6015	1.6673	1.7333	1.7666	1.7866	1.0659	0.4767	0.4472			
-1.6	0.4	0.0519	0.1228	0.2267	0.3989	0.5414	0.6651	1.2284	1.6886	1.7645	1.8414	1.9197	1.9595	1.9837	1.1715	0.5250	0.4482			
-1.6	0.6	0.0578	0.1358	0.2490	0.4344	0.5866	0.7184	1.3214	1.8316	1.9189	2.0088	2.1021	2.1504	2.1800	1.2687	0.5725	0.4512			
-1.6	0.8	0.0629	0.1472	0.2682	0.4647	0.6251	0.7636	1.4019	1.9624	2.0625	2.1675	2.2791	2.3382	2.3749	1.3562	0.6185	0.4560			
-1.6	1.0	0.0672	0.1567	0.2842	0.4898	0.6568	0.8009	1.4695	2.0790	2.1929	2.3150	2.4488	2.5218	2.5680	1.4334	0.6625	0.4622			
-1.6	1.2	0.0707	0.1644	0.2971	0.5098	0.6820	0.8306	1.5244	2.1796	2.3076	2.4485	2.6086	2.6993	2.7583	1.4997	0.7039	0.4693			
-1.6	1.4	0.0735	0.1704	0.3071	0.5253	0.7015	0.8535	1.5675	2.2631	2.4050	2.5653	2.7553	2.8682	2.9445	1.5551	0.7417	0.4770			
-1.6	1.6	0.0756	0.1749	0.3146	0.5368	0.7160	0.8706	1.6000	2.3294	2.4840	2.6632	2.8854	3.0251	3.1244	1.6000	0.7755	0.4847			
-1.6	1.8	0.0771	0.1781	0.3199	0.5451	0.7265	0.8829	1.6237	2.3797	2.5450	2.7414	2.9958	3.1661	3.2952	1.6352	0.8045	0.4920			
-1.6	2.0	0.0782	0.1804	0.3237	0.5508	0.7337	0.8914	1.6402	2.4160	2.5899	2.8004	3.0842	3.2869	3.4525	1.6617	0.8287	0.4987			
-1.6	2.2	0.0789	0.1819	0.3262	0.5546	0.7385	0.8971	1.6513	2.4410	2.6211	2.8425	3.1509	3.3843	3.5914	1.6810	0.8480	0.5044			
-1.6	2.4	0.0793	0.1829	0.3278	0.5571	0.7417	0.9008	1.6584	2.4574	2.6418	2.8708	3.1978	3.4575	3.7068	1.6945	0.8627	0.5091			
-1.6	2.6	0.0796	0.1835	0.3288	0.5586	0.7436	0.9030	1.6629	2.4677	2.6549	2.8889	3.2287	3.5083	3.7954	1.7035	0.8735	0.5128			
-1.6	2.8	0.0798	0.1838	0.3294	0.5595	0.7447	0.9044	1.6655	2.4739	2.6627	2.8999	3.2479	3.5411	3.8577	1.7093	0.8811	0.5155			
-1.6	3.0	0.0799	0.1841	0.3297	0.5601	0.7454	0.9051	1.6670	2.4775	2.6673	2.9062	3.2592	3.5609	3.8977	1.7128	0.8861	0.5173			
-1.4	-1.2	0.0023	0.0057	0.0114	0.0226	0.0336	0.0444	0.1065	0.1639	0.1731	0.1821	0.1911	0.1956	0.1982	0.1043	0.0576	0.5522			
-1.4	-1.0	0.0052	0.0129	0.0256	0.0502	0.0741	0.0973	0.2235	0.3334	0.3505	0.3673	0.3838	0.3919	0.3968	0.2159	0.1145	0.5305			
-1.4	-0.8	0.0087	0.0216	0.0425	0.0827	0.1208	0.1572	0.3476	0.5059	0.5301	0.5539	0.5772	0.5886	0.5955	0.3324	0.1703	0.5125			
-1.4	-0.6	0.0128	0.0316	0.0619	0.1191	0.1723	0.2223	0.4751	0.6793	0.7104	0.7409	0.7707	0.7854	0.7942	0.4517	0.2250	0.4982			
-1.4	-0.4	0.0174	0.0428	0.0832	0.1581	0.2267	0.2901	0.6028	0.8519	0.8899	0.9273	0.9639	0.9820	0.9928	0.5716	0.2786	0.4874			
-1.4	0.0	0.0275	0.0668	0.1282	0.2383	0.3359	0.4243	0.8477	1.1883	1.2417	1.2947	1.3474	1.3737	1.3895	0.8056	0.3829	0.4754			
-1.4	0.2	0.0326	0.0788	0.1502	0.2765	0.3870	0.4864	0.9601	1.3487	1.4112	1.4738	1.5366	1.5682	1.5873	0.9159	0.4339	0.4737			
-1.4	0.4	0.0374	0.0902	0.1708	0.3117	0.4337	0.5429	1.0631	1.5017	1.5744	1.6481	1.7231	1.7613	1.7844	1.0197	0.4840	0.4747			
-1.4	0.6	0.0418	0.1005	0.1893	0.3430	0.4751	0.5927	1.1551	1.6449	1.7292	1.8159	1.9058	1.9523	1.9808	1.1155	0.5330	0.4778			
-1.4	0.8	0.0458	0.1095	0.2054	0.3700	0.5105	0.6353	1.2349	1.7763	1.8734	1.9751	2.0833	2.1404	2.1758	1.2021	0.5804	0.4828			
-1.4	1.0	0.0491	0.1171	0.2189	0.3924	0.5398	0.6706	1.3022	1.8734	1.9751	2.0833	2.1404	2.1758	1.2021	0.5804	0.4828				
-1.4	1.2	0.0518	0.1233	0.2298	0.4104	0.5633	0.6988	1.3570	1.9951	2.1020	2.2578	2.4141	2.5023	2.5596	1.3447	0.6682	0.4969			
-1.4	1.4	0.0539	0.1282	0.2383	0.4243	0.5815	0.7206	1.4000	2.0794	2.1885	2.3757	2.6718	2.7461	1.4000	0.7071	0.5051	0.5051			
-1.4	1.6	0.0555	0.1318	0.2447	0.4347	0.5950	0.7369	1.4325	2.1465	2.2985	2.4748	2.6929	2.8296	2.9265	1.4449	0.7417	0.5134			
-1.4	1.8	0.0567	0.1345	0.2493	0.4422	0.6048	0.7487	1.4562	2.1975	2.3604	2.5539	2.8044	2.9717	3.0979	1.4801	0.7716	0.5213			
-1.4	2.0	0.0575	0.1363	0.2525	0.4474	0.6116	0.7568	1.4728	2.2343	2.4059	2.6139	2.8941	3.0937	3.2562	1.5068	0.7964	0.5286			
-1.4	2.2	0.0581	0.1375	0.2546	0.4509	0.6161	0.7623	1.4839	2.2597	2.4377	2.6567	2.9618	3.1924	3.3962	1.5262	0.8163	0.5349			
-1.4	2.4	0.0584	0.1383	0.2560	0.4532	0.6190	0.7658	1.4911	2.2764	2.4588	2.6855	3.0095	3.2667	3.5128	1.5398	0.8315	0.5400			
-1.4	2.6	0.0586	0.1388	0.2568	0.4546	0.6208	0.7679	1.4955	2.2869	2.4720	2.7040	3.0411	3.3185	3.6027	1.5489	0.8426	0.5440			
-1.4	2.8	0.0588	0.1391	0.2573	0.4554	0.6219	0.7692	1.4982	2.2931	2.4801	2.7152	3.0607	3.3519	3.6661	1.5547	0.8504	0.5470			
-1.4	3.0	0.0588	0.1393	0.2576	0.4559	0.6225	0.7700	1.4997	2.2967	2.4847	2.7217	3.0722	3.3721	3.7069	1.5583	0.8556	0.5491			

Table 1. (continued)

k_u	k_g	Cumulative Probability, $F(t)$																Mean	Std Dev	Cov
		0.01	0.025	0.05	0.1	0.15	0.2	0.5	0.8	0.85	0.9	0.95	0.975	0.99						
-1.2	-1.0	0.0022	0.0056	0.0111	0.0221	0.0330	0.0437	0.1055	0.1633	0.1726	0.1818	0.1910	0.1955	0.1982	0.1037	0.0576	0.5557			
-1.2	-0.8	0.0050	0.0124	0.0246	0.0484	0.0717	0.0943	0.2197	0.3313	0.3488	0.3661	0.3832	0.3916	0.3967	0.2132	0.1147	0.5379			
-1.2	-0.6	0.0082	0.0202	0.0400	0.0783	0.1149	0.1502	0.3392	0.5015	0.5267	0.5515	0.5759	0.5880	0.5952	0.3266	0.1709	0.5235			
-1.2	-0.4	0.0117	0.0290	0.0571	0.1107	0.1614	0.2095	0.4606	0.6718	0.7046	0.7368	0.7686	0.7844	0.7938	0.4415	0.2263	0.5125			
-1.2	-0.2	0.0156	0.0385	0.0753	0.1447	0.2093	0.2701	0.5808	0.8405	0.8811	0.9211	0.9608	0.9804	0.9922	0.5560	0.2807	0.5048			
-1.2	0.0	0.0196	0.0482	0.0938	0.1787	0.2569	0.3297	0.6971	1.0058	1.0548	1.1034	1.1517	1.1759	1.1903	0.6681	0.3342	0.5002			
-1.2	0.2	0.0236	0.0578	0.1119	0.2116	0.3024	0.3863	0.8070	1.1660	1.2241	1.2824	1.3410	1.3704	1.3881	0.7759	0.3869	0.4986			
-1.2	0.4	0.0274	0.0669	0.1290	0.2423	0.3444	0.4384	0.9083	1.3190	1.3875	1.4569	1.5276	1.5636	1.5854	0.8778	0.4386	0.4996			
-1.2	0.6	0.0309	0.0752	0.1445	0.2698	0.3820	0.4847	0.9992	1.4627	1.5429	1.6253	1.7107	1.7548	1.7818	0.9723	0.4891	0.5030			
-1.2	0.8	0.0340	0.0825	0.1581	0.2937	0.4143	0.5245	1.0784	1.5949	1.6879	1.7853	1.8887	1.9432	1.9770	1.0581	0.5379	0.5084			
-1.2	1.0	0.0366	0.0887	0.1695	0.3136	0.4413	0.5577	1.1453	1.7132	1.8201	1.9346	2.0597	2.1276	2.1704	1.1342	0.5845	0.5154			
-1.2	1.2	0.0387	0.0938	0.1787	0.3297	0.4630	0.5843	1.2000	1.8157	1.9370	2.0703	2.2133	2.3062	2.3613	1.2000	0.6282	0.5235			
-1.2	1.4	0.0404	0.0977	0.1859	0.3422	0.4798	0.6049	1.2430	1.9012	2.0367	2.1896	2.3702	2.4767	2.5482	1.2553	0.6682	0.5323			
-1.2	1.6	0.0417	0.1007	0.1914	0.3515	0.4924	0.6204	1.2756	1.9694	2.1180	2.2902	2.5029	2.6356	2.7293	1.3003	0.7039	0.5413			
-1.2	1.8	0.0426	0.1029	0.1953	0.3583	0.5015	0.6316	1.2994	2.0213	2.1811	2.3708	2.6161	2.7791	2.9015	1.3357	0.7347	0.5500			
-1.2	2.0	0.0433	0.1044	0.1980	0.3630	0.5078	0.6393	1.3160	2.0589	2.2276	2.4320	2.7074	2.9028	3.0610	1.3626	0.7603	0.5580			
-1.2	2.2	0.0437	0.1054	0.1999	0.3661	0.5120	0.6445	1.3271	2.0849	2.2601	2.4758	2.7765	3.0032	3.2025	1.3822	0.7808	0.5649			
-1.2	2.4	0.0440	0.1060	0.2010	0.3681	0.5147	0.6478	1.3343	2.1019	2.2816	2.5054	2.8254	3.0791	3.3209	1.3959	0.7966	0.5706			
-1.2	2.6	0.0442	0.1065	0.2018	0.3694	0.5164	0.6499	1.3388	2.1126	2.2953	2.5244	2.8578	3.1322	3.4126	1.4052	0.8081	0.5751			
-1.2	2.8	0.0443	0.1067	0.2022	0.3701	0.5174	0.6511	1.3415	2.1191	2.3035	2.5359	2.8780	3.1666	3.4775	1.4111	0.8162	0.5784			
-1.2	3.0	0.0443	0.1068	0.2025	0.3706	0.5180	0.6518	1.3430	2.1228	2.3082	2.5426	2.8898	3.1874	3.5194	1.4148	0.8216	0.5808			
-1.0	-0.8	0.0022	0.0055	0.0109	0.0218	0.0325	0.0430	0.1045	0.1627	0.1721	0.1815	0.1908	0.1954	0.1982	0.1030	0.0576	0.5595			
-1.0	-0.6	0.0048	0.0119	0.0236	0.0467	0.0693	0.0914	0.2158	0.3291	0.3471	0.3649	0.3826	0.3913	0.3965	0.2106	0.1149	0.5455			
-1.0	-0.4	0.0077	0.0190	0.0377	0.0741	0.1093	0.1434	0.3306	0.4967	0.5230	0.5490	0.5746	0.5873	0.5949	0.3207	0.1714	0.5346			
-1.0	-0.2	0.0108	0.0267	0.0528	0.1030	0.1511	0.1973	0.4457	0.6637	0.6983	0.7325	0.7664	0.7832	0.7933	0.4312	0.2272	0.5270			
-1.0	0.0	0.0140	0.0347	0.0682	0.1323	0.1931	0.2510	0.5582	0.8280	0.8713	0.9143	0.9572	0.9786	0.9914	0.5401	0.2822	0.5225			
-1.0	0.2	0.0172	0.0426	0.0834	0.1609	0.2337	0.3026	0.6654	0.9878	1.0405	1.0934	1.1465	1.1732	1.1893	0.6456	0.3364	0.5210			
-1.0	0.4	0.0203	0.0501	0.0979	0.1878	0.2716	0.3505	0.7648	1.1410	1.2042	1.2682	1.3334	1.3665	1.3865	0.7458	0.3895	0.5223			
-1.0	0.6	0.0232	0.0570	0.1110	0.2121	0.3056	0.3934	0.8546	1.2854	1.3603	1.4373	1.5169	1.5580	1.5831	0.8391	0.4414	0.5261			
-1.0	0.8	0.0257	0.0631	0.1226	0.2332	0.3352	0.4306	0.9333	1.4186	1.5064	1.5983	1.6956	1.7468	1.7785	0.9242	0.4916	0.5320			
-1.0	1.0	0.0278	0.0682	0.1323	0.2510	0.3599	0.4617	1.0000	1.5383	1.6401	1.7490	1.8677	1.9318	1.9722	1.0000	0.5396	0.5396			
-1.0	1.2	0.0296	0.0724	0.1403	0.2654	0.3799	0.4868	1.0547	1.6423	1.7587	1.8864	2.0305	2.1113	2.1634	1.0658	0.5845	0.5484			
-1.0	1.4	0.0310	0.0757	0.1465	0.2766	0.3955	0.5063	1.0978	1.7294	1.8602	2.0076	2.1811	2.2829	2.3509	1.1213	0.6257	0.5581			
-1.0	1.6	0.0320	0.0782	0.1512	0.2850	0.4071	0.5210	1.1305	1.7991	1.9432	2.1102	2.3158	2.4433	2.5328	1.1666	0.6625	0.5679			
-1.0	1.8	0.0327	0.0800	0.1545	0.2911	0.4156	0.5316	1.1544	1.8523	2.0079	2.1928	2.4311	2.5887	2.7062	1.2024	0.6944	0.5775			
-1.0	2.0	0.0333	0.0813	0.1569	0.2954	0.4214	0.5389	1.1712	1.8910	2.0558	2.2527	2.5246	2.7145	2.8672	1.2296	0.7209	0.5863			
-1.0	2.2	0.0336	0.0821	0.1585	0.2982	0.4254	0.5439	1.1824	1.9177	2.0893	2.3009	2.5957	2.8173	3.0107	1.2496	0.7423	0.5940			
-1.0	2.4	0.0339	0.0827	0.1595	0.3000	0.4279	0.5470	1.1897	1.9353	2.1115	2.3315	2.6463	2.8953	3.1314	1.2636	0.7587	0.6004			
-1.0	2.6	0.0340	0.0830	0.1601	0.3011	0.4295	0.5490	1.1942	1.9463	2.1256	2.3511	2.6798	2.9502	3.2254	1.2730	0.7708	0.6055			
-1.0	2.8	0.0341	0.0832	0.1605	0.3018	0.4304	0.5500	1.1969	1.9529	2.1341	2.3630	2.7008	2.9858	3.2923	1.2790	0.7793	0.6093			
-1.0	3.0	0.0341	0.0833	0.1607	0.3022	0.4309	0.5508	1.1984	1.9567	2.1390	2.3700	2.7131	3.0075	3.3358	1.2828	0.7849	0.6119			

Table 1. (continued)

k_d	k_R	0.01	0.025	0.05	0.1	0.15	0.2	0.5	0.8	0.85	0.9	0.95	0.975	0.99	Mean	Std Dev	CoV
-0.8	-0.6	0.0022	0.0054	0.0107	0.0214	0.0319	0.0424	0.1035	0.1621	0.1717	0.1812	0.1906	0.1953	0.1981	1.023	0.0577	0.5636
-0.8	-0.4	0.0046	0.0114	0.0227	0.0450	0.0670	0.0885	0.2119	0.3268	0.3453	0.3637	0.3819	0.3910	0.3964	2.080	0.1150	0.5530
-0.8	-0.2	0.0072	0.0179	0.0355	0.0702	0.1039	0.1369	0.3219	0.4918	0.5191	0.5463	0.5732	0.5866	0.5947	3.148	0.1718	0.5457
-0.8	0.0	0.0099	0.0246	0.0488	0.0959	0.1414	0.1856	0.4306	0.6550	0.6914	0.7277	0.7639	0.7819	0.7928	4.208	0.2279	0.5416
-0.8	0.2	0.0126	0.0313	0.0619	0.1211	0.1780	0.2329	0.5351	0.8145	0.8606	0.9068	0.9532	0.9766	0.9906	5.241	0.2833	0.5405
-0.8	0.4	0.0152	0.0377	0.0744	0.1449	0.2124	0.2772	0.6329	0.9680	1.0247	1.0821	1.1405	1.1701	1.1880	6.228	0.3377	0.5422
-0.8	0.6	0.0176	0.0436	0.0858	0.1666	0.2435	0.3172	0.7217	1.1133	1.1817	1.2520	1.3246	1.3619	1.3847	7.153	0.3909	0.5465
-0.8	0.8	0.0197	0.0488	0.0959	0.1856	0.2707	0.3521	0.8000	1.2479	1.3293	1.4144	1.5042	1.5512	1.5803	8.000	0.4424	0.5530
-0.8	1.0	0.0215	0.0532	0.1044	0.2017	0.2936	0.3814	0.8667	1.3694	1.4648	1.5668	1.6774	1.7370	1.7743	8.758	0.4916	0.5613
-0.8	1.2	0.0230	0.0568	0.1113	0.2147	0.3121	0.4051	0.9216	1.4755	1.5857	1.7063	1.8419	1.9175	1.9660	9.419	0.5379	0.5711
-0.8	1.4	0.0242	0.0596	0.1168	0.2248	0.3266	0.4237	0.9651	1.5647	1.6895	1.8300	1.9946	2.0905	2.1542	9.979	0.5804	0.5817
-0.8	1.6	0.0251	0.0618	0.1209	0.2325	0.3375	0.4376	0.9981	1.6364	1.7749	1.9353	2.1318	2.2528	2.3371	1.0438	0.6185	0.5926
-0.8	1.8	0.0257	0.0633	0.1238	0.2381	0.3454	0.4477	1.0223	1.6913	1.8418	2.0204	2.2499	2.4006	2.5120	1.0802	0.6515	0.6032
-0.8	2.0	0.0261	0.0644	0.1259	0.2419	0.3509	0.4547	1.0393	1.7314	1.8914	2.0856	2.3463	2.5292	2.6749	1.1079	0.6792	0.6130
-0.8	2.2	0.0264	0.0651	0.1273	0.2445	0.3545	0.4594	1.0507	1.7591	1.9262	2.1326	2.4200	2.6349	2.8209	1.1283	0.7015	0.6217
-0.8	2.4	0.0266	0.0656	0.1282	0.2462	0.3569	0.4624	1.0581	1.7774	1.9494	2.1646	2.4727	2.7158	2.9445	1.1427	0.7187	0.6289
-0.8	2.6	0.0268	0.0659	0.1288	0.2472	0.3584	0.4643	1.0627	1.7889	1.9641	2.1851	2.5078	2.7730	3.0415	1.1524	0.7314	0.6347
-0.8	2.8	0.0268	0.0661	0.1291	0.2478	0.3592	0.4654	1.0654	1.7958	1.9730	2.1976	2.5298	2.8104	3.1112	1.1587	0.7404	0.6390
-0.8	3.0	0.0269	0.0662	0.1293	0.2482	0.3597	0.4661	1.0670	1.7998	1.9781	2.2049	2.5427	2.8331	3.1567	1.1626	0.7464	0.6420
-0.6	-0.4	0.0021	0.0053	0.0105	0.0210	0.0314	0.0417	0.1025	0.1615	0.1712	0.1808	0.1904	0.1952	0.1981	1.017	0.0577	0.5675
-0.6	-0.2	0.0044	0.0110	0.0218	0.0434	0.0647	0.0858	0.2079	0.3244	0.3435	0.3624	0.3812	0.3906	0.3963	2.053	0.1151	0.5606
-0.6	0.0	0.0068	0.0169	0.0335	0.0665	0.0988	0.1306	0.3132	0.4866	0.5150	0.5434	0.5717	0.5859	0.5943	3.089	0.1720	0.5569
-0.6	0.2	0.0091	0.0227	0.0452	0.0892	0.1323	0.1745	0.4154	0.6458	0.6841	0.7225	0.7611	0.7805	0.7922	4.104	0.2283	0.5563
-0.6	0.4	0.0114	0.0284	0.0563	0.1109	0.1640	0.2160	0.5117	0.7998	0.8488	0.8984	0.9488	0.9743	0.9897	5.081	0.2838	0.5586
-0.6	0.6	0.0135	0.0335	0.0665	0.1306	0.1929	0.2537	0.6000	0.9463	1.0071	1.0694	1.1335	1.1665	1.1865	6.000	0.3381	0.5636
-0.6	0.8	0.0154	0.0381	0.0754	0.1480	0.2183	0.2867	0.6783	1.0828	1.1565	1.2334	1.3142	1.3564	1.3824	6.847	0.3909	0.5709
-0.6	1.0	0.0169	0.0420	0.0831	0.1627	0.2397	0.3146	0.7454	1.2066	1.2944	1.3879	1.4890	1.5430	1.5768	7.609	0.4414	0.5801
-0.6	1.2	0.0182	0.0452	0.0893	0.1747	0.2571	0.3373	0.8008	1.3153	1.4180	1.5302	1.6555	1.7248	1.7691	8.277	0.4891	0.5909
-0.6	1.4	0.0192	0.0477	0.0942	0.1841	0.2708	0.3551	0.8449	1.4073	1.5249	1.6570	1.8107	1.8995	1.9582	8.845	0.5330	0.6026
-0.6	1.6	0.0200	0.0496	0.0979	0.1912	0.2811	0.3684	0.8786	1.4816	1.6134	1.7656	1.9510	2.0642	2.1422	9.313	0.5725	0.6147
-0.6	1.8	0.0206	0.0510	0.1006	0.1964	0.2885	0.3781	0.9033	1.5389	1.6830	1.8541	2.0727	2.2148	2.3188	9.686	0.6069	0.6265
-0.6	2.0	0.0210	0.0519	0.1025	0.1999	0.2937	0.3849	0.9206	1.5808	1.7349	1.9222	2.1727	2.3469	2.4840	9.972	0.6358	0.6376
-0.6	2.2	0.0212	0.0526	0.1037	0.2023	0.2972	0.3895	0.9323	1.6099	1.7716	1.9716	2.2498	2.4564	2.6331	1.0183	0.6592	0.6474
-0.6	2.4	0.0214	0.0530	0.1045	0.2039	0.2995	0.3924	0.9399	1.6291	1.7961	2.0054	2.3052	2.5409	2.7603	1.0332	0.6774	0.6556
-0.6	2.6	0.0215	0.0533	0.1050	0.2048	0.3009	0.3942	0.9446	1.6413	1.8116	2.0271	2.3424	2.6011	2.8612	1.0433	0.6908	0.6622
-0.6	2.8	0.0216	0.0534	0.1053	0.2054	0.3017	0.3953	0.9474	1.6485	1.8210	2.0404	2.3658	2.6408	2.9344	1.0498	0.7004	0.6672
-0.6	3.0	0.0216	0.0535	0.1055	0.2057	0.3022	0.3959	0.9490	1.6527	1.8265	2.0481	2.3796	2.6650	2.9827	1.0539	0.7068	0.6707

Table 1. (continued)

k_c	k_b	0.01	0.025	0.05	0.1	0.15	0.2	0.5	0.85	0.9	0.95	0.975	0.99	Mean	Std Dev	Cov	
		Cumulative Probability, $F(t)$															
-0.4	-0.2	0.0021	0.0052	0.0103	0.0206	0.0308	0.0410	0.1015	0.1609	0.1707	0.1805	0.1902	0.1951	0.1981	0.1010	0.0577	0.5712
-0.4	0.0	0.0042	0.0105	0.0210	0.0419	0.0626	0.0831	0.2040	0.3200	0.3415	0.3610	0.3805	0.3903	0.3961	0.2027	0.1151	0.5682
-0.4	0.2	0.0064	0.0159	0.0317	0.0630	0.0939	0.1246	0.3044	0.4811	0.5107	0.5403	0.5701	0.5850	0.5940	0.3030	0.1722	0.5682
-0.4	0.4	0.0084	0.0210	0.0419	0.0831	0.1238	0.1641	0.4000	0.6359	0.6762	0.7169	0.7581	0.7790	0.7916	0.4000	0.2285	0.5712
-0.4	0.6	0.0103	0.0257	0.0512	0.1016	0.1512	0.2002	0.4883	0.7840	0.8360	0.8891	0.9437	0.9716	0.9886	0.4919	0.2838	0.5769
-0.4	0.8	0.0120	0.0299	0.0595	0.1179	0.1753	0.2320	0.5671	0.9228	0.9876	1.0551	1.1256	1.1623	1.1848	0.5772	0.3377	0.5851
-0.4	1.0	0.0135	0.0335	0.0666	0.1318	0.1958	0.2590	0.6352	1.0495	1.1284	1.2122	1.3021	1.3499	1.3797	0.6542	0.3895	0.5954
-0.4	1.2	0.0146	0.0364	0.0724	0.1431	0.2125	0.2810	0.6917	1.1616	1.2556	1.3577	1.4710	1.5331	1.5726	0.7222	0.4386	0.6073
-0.4	1.4	0.0156	0.0387	0.0769	0.1519	0.2256	0.2983	0.7369	1.2571	1.3663	1.4883	1.6292	1.7098	1.7626	0.7803	0.4840	0.6203
-0.4	1.6	0.0163	0.0405	0.0803	0.1586	0.2355	0.3114	0.7716	1.3349	1.4586	1.6011	1.7733	1.8772	1.9480	0.8285	0.5250	0.6338
-0.4	1.8	0.0168	0.0417	0.0828	0.1635	0.2427	0.3209	0.7970	1.3952	1.5318	1.6937	1.8992	2.0313	2.1265	0.8670	0.5610	0.6470
-0.4	2.0	0.0171	0.0426	0.0846	0.1669	0.2477	0.3275	0.8150	1.4395	1.5868	1.7657	2.0038	2.1676	2.2945	0.8967	0.5914	0.6595
-0.4	2.2	0.0174	0.0432	0.0857	0.1692	0.2511	0.3320	0.8271	1.4705	1.6258	1.8183	2.0852	2.2817	2.4472	0.9188	0.6161	0.6706
-0.4	2.4	0.0175	0.0436	0.0865	0.1707	0.2532	0.3348	0.8349	1.4910	1.6520	1.8544	2.1442	2.3707	2.5789	0.9344	0.6354	0.6800
-0.4	2.6	0.0176	0.0438	0.0869	0.1716	0.2546	0.3366	0.8398	1.5039	1.6687	1.8777	2.1842	2.4349	2.6846	0.9450	0.6499	0.6876
-0.4	2.8	0.0177	0.0439	0.0872	0.1721	0.2554	0.3377	0.8427	1.5117	1.6788	1.8921	2.2093	2.4775	2.7622	0.9520	0.6601	0.6934
-0.4	3.0	0.0177	0.0440	0.0874	0.1724	0.2558	0.3383	0.8444	1.5162	1.6846	1.9004	2.2243	2.5037	2.8139	0.9563	0.6671	0.6976
-0.2	0.0	0.0020	0.0051	0.0101	0.0202	0.0303	0.0404	0.1005	0.1603	0.1702	0.1801	0.1901	0.1950	0.1980	0.1003	0.0577	0.5750
-0.2	0.2	0.0041	0.0101	0.0202	0.0404	0.0605	0.0805	0.2000	0.3195	0.3395	0.3596	0.3798	0.3899	0.3959	0.2000	0.1152	0.5758
-0.2	0.4	0.0060	0.0150	0.0299	0.0597	0.0893	0.1189	0.2956	0.4754	0.5061	0.5370	0.5683	0.5841	0.5936	0.2970	0.1722	0.5796
-0.2	0.6	0.0078	0.0195	0.0389	0.0775	0.1159	0.1542	0.3846	0.6255	0.6677	0.7108	0.7548	0.7773	0.7909	0.3896	0.2283	0.5861
-0.2	0.8	0.0094	0.0234	0.0468	0.0932	0.1394	0.1855	0.4649	0.7671	0.8220	0.8789	0.9381	0.9687	0.9874	0.4759	0.2833	0.5953
-0.2	1.0	0.0107	0.0268	0.0536	0.1066	0.1595	0.2122	0.5346	0.8974	0.9663	1.0391	1.1166	1.1574	1.1828	0.5544	0.3364	0.6067
-0.2	1.2	0.0119	0.0296	0.0590	0.1176	0.1759	0.2340	0.5930	1.0137	1.0976	1.1884	1.2881	1.3423	1.3764	0.6241	0.3869	0.6199
-0.2	1.4	0.0127	0.0318	0.0634	0.1262	0.1888	0.2513	0.6399	1.1136	1.2130	1.3235	1.4498	1.5212	1.5675	0.6841	0.4339	0.6343
-0.2	1.6	0.0134	0.0334	0.0667	0.1327	0.1985	0.2643	0.6760	1.1957	1.3101	1.4413	1.5982	1.6915	1.7544	0.7341	0.4767	0.6493
-0.2	1.8	0.0139	0.0346	0.0690	0.1375	0.2056	0.2738	0.7027	1.2599	1.3879	1.5391	1.7293	1.8496	1.9351	0.7744	0.5144	0.6642
-0.2	2.0	0.0142	0.0355	0.0707	0.1408	0.2106	0.2804	0.7215	1.3074	1.4469	1.6159	1.8393	1.9908	2.1062	0.8057	0.5465	0.6783
-0.2	2.2	0.0144	0.0360	0.0718	0.1430	0.2139	0.2849	0.7343	1.3408	1.4890	1.6725	1.9260	2.1105	2.2630	0.8289	0.5728	0.6910
-0.2	2.4	0.0146	0.0364	0.0726	0.1444	0.2160	0.2877	0.7425	1.3630	1.5174	1.7117	1.9897	2.2051	2.3999	0.8455	0.5935	0.7019
-0.2	2.6	0.0147	0.0366	0.0730	0.1453	0.2174	0.2895	0.7477	1.3771	1.5355	1.7372	2.0331	2.2743	2.5113	0.8569	0.6090	0.7107
-0.2	2.8	0.0147	0.0367	0.0733	0.1459	0.2182	0.2906	0.7507	1.3856	1.5466	1.7529	2.0607	2.3207	2.5945	0.8643	0.6202	0.7175
-0.2	3.0	0.0148	0.0368	0.0734	0.1462	0.2186	0.2912	0.7525	1.3905	1.5529	1.7620	2.0772	2.3495	2.6507	0.8690	0.6278	0.7225
0.0	0.2	0.0020	0.0050	0.0099	0.0199	0.0298	0.0397	0.0995	0.1596	0.1697	0.1798	0.1899	0.1949	0.1980	0.0997	0.0577	0.5789
0.0	0.4	0.0039	0.0097	0.0195	0.0390	0.0585	0.0780	0.1960	0.3169	0.3374	0.3581	0.3790	0.3895	0.3958	0.1973	0.1151	0.5835
0.0	0.6	0.0057	0.0141	0.0283	0.0566	0.0850	0.1134	0.2868	0.4694	0.5012	0.5335	0.5665	0.5831	0.5932	0.2911	0.1720	0.5909
0.0	0.8	0.0072	0.0181	0.0361	0.0723	0.1086	0.1450	0.3694	0.6144	0.6586	0.7042	0.7512	0.7754	0.7901	0.3792	0.2279	0.6011
0.0	1.0	0.0086	0.0214	0.0428	0.0857	0.1287	0.1720	0.4418	0.7490	0.8069	0.8677	0.9318	0.9653	0.9860	0.4599	0.2822	0.6137
0.0	1.2	0.0097	0.0241	0.0483	0.0966	0.1452	0.1942	0.5029	0.8703	0.9431	1.0213	1.1062	1.1518	1.1804	0.5319	0.3342	0.6283
0.0	1.4	0.0105	0.0263	0.0526	0.1053	0.1583	0.2117	0.5523	0.9757	1.0641	1.1617	1.2718	1.3332	1.3725	0.5944	0.3829	0.6442
0.0	1.6	0.0112	0.0279	0.0558	0.1118	0.1682	0.2251	0.5906	1.0632	1.1671	1.2854	1.4251	1.5068	1.5611	0.6469	0.4276	0.6609
0.0	1.8	0.0116	0.0291	0.0582	0.1166	0.1754	0.2348	0.6190	1.1324	1.2506	1.3895	1.5621	1.6693	1.7441	0.6895	0.4672	0.6776
0.0	2.0	0.0120	0.0299	0.0599	0.1199	0.1804	0.2416	0.6391	1.1840	1.3145	1.4723	1.6786	1.8161	1.9185	0.7228	0.5013	0.6936
0.0	2.2	0.0122	0.0305	0.0610	0.1221	0.1838	0.2462	0.6528	1.2206	1.3606	1.5340	1.7719	1.9422	2.0800	0.7477	0.5295	0.7081
0.0	2.4	0.0123	0.0308	0.0617	0.1236	0.1860	0.2491	0.6616	1.2450	1.3920	1.5772	1.8413	2.0437	2.2228	0.7656	0.5518	0.7207
0.0	2.6	0.0124	0.0310	0.0621	0.1245	0.1873	0.2509	0.6672	1.2606	1.4121	1.6055	1.8893	2.1190	2.3411	0.7780	0.5687	0.7311
0.0	2.8	0.0125	0.0312	0.0624	0.1250	0.1881	0.2520	0.6700	1.2700	1.4244	1.6230	1.9200	2.1703	2.4310	0.7861	0.5810	0.7391
0.0	3.0	0.0125	0.0313	0.0625	0.1253	0.1886	0.2526	0.6724	1.2754	1.4315	1.6332	1.9385	2.2025	2.4928	0.7912	0.5894	0.7450

Table 1. (continued)

k_p	k_g	Cumulative Probability, $F(t)$																Mean	Std Dev	CoV
		0.01	0.025	0.05	0.1	0.15	0.2	0.5	0.8	0.85	0.9	0.95	0.975	0.99						
0.2	0.4	0.0019	0.0049	0.0097	0.0195	0.0293	0.0391	0.0985	0.1590	0.1692	0.1794	0.1897	0.1948	0.1979	0.0990	0.0577	0.5827			
0.2	0.6	0.0037	0.0094	0.0188	0.0376	0.0565	0.0756	0.1921	0.3142	0.3353	0.3566	0.3782	0.3890	0.3956	0.1947	0.1151	0.5911			
0.2	0.8	0.0053	0.0134	0.0268	0.0537	0.0809	0.1082	0.2781	0.4631	0.4961	0.5298	0.5645	0.5821	0.5928	0.2852	0.1718	0.6024			
0.2	1.0	0.0067	0.0168	0.0336	0.0675	0.1017	0.1363	0.3543	0.6027	0.6489	0.6970	0.7472	0.7733	0.7892	0.3688	0.2272	0.6161			
0.2	1.2	0.0078	0.0196	0.0392	0.0789	0.1189	0.1595	0.4192	0.7299	0.7907	0.8553	0.9247	0.9615	0.9844	0.4440	0.2807	0.6321			
0.2	1.4	0.0087	0.0218	0.0437	0.0878	0.1325	0.1779	0.4721	0.8418	0.9181	1.0016	1.0944	1.1453	1.1776	0.5098	0.3312	0.6497			
0.2	1.6	0.0094	0.0235	0.0470	0.0946	0.1429	0.1919	0.5135	0.9360	1.0282	1.1322	1.2531	1.3224	1.3678	0.5655	0.3779	0.6682			
0.2	1.8	0.0098	0.0247	0.0495	0.0996	0.1504	0.2022	0.5443	1.0113	1.1186	1.2437	1.3967	1.4896	1.5533	0.6110	0.4197	0.6869			
0.2	2.0	0.0102	0.0255	0.0512	0.1030	0.1557	0.2093	0.5663	1.0683	1.1888	1.3337	1.5207	1.6425	1.7312	0.6469	0.4560	0.7050			
0.2	2.2	0.0104	0.0261	0.0523	0.1053	0.1592	0.2142	0.5813	1.1090	1.2400	1.4019	1.6218	1.7761	1.8976	0.6740	0.4864	0.7216			
0.2	2.4	0.0106	0.0264	0.0531	0.1068	0.1615	0.2173	0.5910	1.1364	1.2752	1.4502	1.6984	1.8855	2.0469	0.6936	0.5107	0.7362			
0.2	2.6	0.0107	0.0267	0.0535	0.1078	0.1629	0.2192	0.5971	1.1539	1.2980	1.4822	1.7521	1.9684	2.1730	0.7072	0.5293	0.7484			
0.2	2.8	0.0107	0.0268	0.0538	0.1083	0.1638	0.2204	0.6007	1.1646	1.3119	1.5020	1.7869	2.0259	2.2710	0.7162	0.5428	0.7580			
0.2	3.0	0.0107	0.0269	0.0539	0.1086	0.1643	0.2210	0.6028	1.1707	1.3200	1.5137	1.8080	2.0625	2.3399	0.7218	0.5523	0.7651			
0.4	0.6	0.0019	0.0048	0.0096	0.0192	0.0288	0.0385	0.0975	0.1583	0.1686	0.1790	0.1895	0.1947	0.1979	0.0983	0.0577	0.5867			
0.4	0.8	0.0036	0.0090	0.0181	0.0363	0.0547	0.0732	0.1881	0.3115	0.3331	0.3550	0.3773	0.3886	0.3954	0.1920	0.1150	0.5988			
0.4	1.0	0.0051	0.0127	0.0254	0.0510	0.0770	0.1033	0.2694	0.4566	0.4907	0.5259	0.5623	0.5810	0.5923	0.2793	0.1714	0.6138			
0.4	1.2	0.0062	0.0156	0.0314	0.0631	0.0954	0.1282	0.3394	0.5905	0.6386	0.6893	0.7429	0.7710	0.7883	0.3585	0.2263	0.6311			
0.4	1.4	0.0072	0.0180	0.0361	0.0727	0.1101	0.1481	0.3972	0.7099	0.7733	0.8419	0.9168	0.9572	0.9826	0.4284	0.2786	0.6504			
0.4	1.6	0.0079	0.0198	0.0397	0.0800	0.1212	0.1633	0.4427	0.8120	0.8915	0.9801	1.0811	1.1378	1.1744	0.4881	0.3274	0.6708			
0.4	1.8	0.0084	0.0210	0.0423	0.0854	0.1294	0.1744	0.4770	0.8950	0.9902	1.001	1.2318	1.3098	1.3622	0.5374	0.3717	0.6917			
0.4	2.0	0.0088	0.0219	0.0441	0.0891	0.1351	0.1822	0.5015	0.9587	1.0682	1.1988	1.3643	1.4692	1.5437	0.5765	0.4106	0.7121			
0.4	2.2	0.0090	0.0226	0.0453	0.0916	0.1389	0.1875	0.5183	1.0048	1.1260	1.2750	1.4744	1.6109	1.7151	0.6064	0.4434	0.7312			
0.4	2.4	0.0092	0.0229	0.0461	0.0932	0.1414	0.1909	0.5292	1.0362	1.1662	1.3298	1.5597	1.7295	1.8714	0.6282	0.4700	0.7481			
0.4	2.6	0.0092	0.0232	0.0466	0.0942	0.1429	0.1930	0.5361	1.0564	1.1924	1.3665	1.6207	1.8215	2.0062	0.6435	0.4906	0.7624			
0.4	2.8	0.0093	0.0233	0.0469	0.0948	0.1438	0.1942	0.5402	1.0687	1.2085	1.3896	1.6609	1.8867	2.1136	0.6536	0.5058	0.7739			
0.4	3.0	0.0093	0.0234	0.0471	0.0951	0.1444	0.1950	0.5425	1.0758	1.2180	1.4033	1.6855	1.9290	2.1912	0.6600	0.5165	0.7825			
0.6	0.8	0.0019	0.0047	0.0094	0.0188	0.0283	0.0379	0.0965	0.1576	0.1681	0.1786	0.1893	0.1946	0.1978	0.0977	0.0577	0.5905			
0.6	1.0	0.0035	0.0087	0.0174	0.0351	0.0529	0.0709	0.1842	0.3086	0.3307	0.3533	0.3764	0.3881	0.3952	0.1894	0.1149	0.6065			
0.6	1.2	0.0048	0.0120	0.0241	0.0485	0.0733	0.0985	0.2608	0.4498	0.4851	0.5217	0.5600	0.5798	0.5918	0.2734	0.1709	0.6251			
0.6	1.4	0.0058	0.0146	0.0293	0.0591	0.0896	0.1207	0.3249	0.5777	0.6277	0.6809	0.7381	0.7684	0.7872	0.3483	0.2250	0.6460			
0.6	1.6	0.0066	0.0165	0.0333	0.0672	0.1020	0.1377	0.3760	0.6891	0.7549	0.8273	0.9080	0.9524	0.9805	0.4130	0.2761	0.6684			
0.6	1.8	0.0072	0.0180	0.0362	0.0732	0.1111	0.1501	0.4148	0.7813	0.8634	0.9568	1.0661	1.1292	1.1706	0.4669	0.3229	0.6916			
0.6	2.0	0.0076	0.0190	0.0382	0.0773	0.1175	0.1589	0.4429	0.8534	0.9508	1.0656	1.2077	1.2951	1.3555	0.5103	0.3646	0.7145			
0.6	2.2	0.0078	0.0197	0.0395	0.0801	0.1218	0.1648	0.4622	0.9065	1.0168	1.1514	1.3281	1.4453	1.5319	0.5437	0.4003	0.7363			
0.6	2.4	0.0080	0.0201	0.0404	0.0819	0.1246	0.1687	0.4748	0.9431	1.0635	1.2145	1.4237	1.5741	1.6953	0.5683	0.4296	0.7560			
0.6	2.6	0.0081	0.0204	0.0410	0.0830	0.1263	0.1711	0.4828	0.9668	1.0943	1.2575	1.4938	1.6767	1.8393	0.5857	0.4526	0.7728			
0.6	2.8	0.0082	0.0205	0.0413	0.0837	0.1274	0.1725	0.4875	0.9814	1.1135	1.2848	1.5409	1.7516	1.9575	0.5973	0.4698	0.7865			
0.6	3.0	0.0082	0.0206	0.0415	0.0841	0.1280	0.1733	0.4903	0.9899	1.1248	1.3012	1.5702	1.8011	2.0455	0.6048	0.4820	0.7970			

Table 1. (continued)

k_L	k_R	Cumulative Probability, $F(t)$																	Mean	Std Dev	CoV
		0.01	0.025	0.05	0.1	0.15	0.2	0.5	0.8	0.85	0.9	0.95	0.975	0.99							
0.8	1.0	0.0018	0.0046	0.0092	0.0185	0.0279	0.0373	0.0955	0.1570	0.1675	0.1782	0.1891	0.1945	0.1978	0.0970	0.0576	0.5941				
0.8	1.2	0.0033	0.0084	0.0168	0.0339	0.0512	0.0687	0.1803	0.3057	0.3283	0.3516	0.3754	0.3876	0.3950	0.1868	0.1147	0.6141				
0.8	1.4	0.0045	0.0114	0.0228	0.0461	0.0699	0.0941	0.2524	0.4428	0.4792	0.5173	0.5575	0.5784	0.5913	0.2676	0.1703	0.6365				
0.8	1.6	0.0054	0.0136	0.0274	0.0555	0.0842	0.1137	0.3108	0.5645	0.6162	0.6720	0.7329	0.7656	0.7860	0.3383	0.2235	0.6609				
0.8	1.8	0.0061	0.0153	0.0307	0.0623	0.0947	0.1281	0.3557	0.6675	0.7356	0.8116	0.8983	0.9469	0.9781	0.3979	0.2731	0.6863				
0.8	2.0	0.0065	0.0164	0.0331	0.0671	0.1021	0.1383	0.3886	0.7500	0.8341	0.9317	1.0493	1.1192	1.1661	0.4464	0.3178	0.7119				
0.8	2.2	0.0069	0.0172	0.0346	0.0703	0.1072	0.1453	0.4114	0.8119	0.9104	1.0290	1.1809	1.2780	1.3473	0.4842	0.3567	0.7366				
0.8	2.4	0.0071	0.0177	0.0357	0.0724	0.1104	0.1497	0.4264	0.8553	0.9654	1.1023	1.2884	1.4177	1.5177	0.5125	0.3892	0.7594				
0.8	2.6	0.0072	0.0180	0.0363	0.0737	0.1124	0.1525	0.4358	0.8839	1.0024	1.1535	1.3695	1.5323	1.6712	0.5326	0.4150	0.7792				
0.8	2.8	0.0072	0.0182	0.0367	0.0745	0.1136	0.1542	0.4415	0.9016	1.0257	1.1866	1.4256	1.6187	1.8010	0.5463	0.4347	0.7957				
0.8	3.0	0.0073	0.0183	0.0369	0.0749	0.1143	0.1551	0.4448	0.9120	1.0394	1.2065	1.4611	1.6777	1.9012	0.5551	0.4488	0.8085				
1.0	1.2	0.0018	0.0045	0.0091	0.0182	0.0274	0.0367	0.0945	0.1563	0.1670	0.1779	0.1888	0.1944	0.1978	0.0963	0.0576	0.5979				
1.0	1.4	0.0032	0.0081	0.0162	0.0327	0.0495	0.0666	0.1765	0.3027	0.3259	0.3498	0.3744	0.3871	0.3948	0.1841	0.1145	0.6218				
1.0	1.6	0.0043	0.0108	0.0217	0.0439	0.0666	0.0899	0.2441	0.4355	0.4730	0.5126	0.5548	0.5770	0.5907	0.2618	0.1696	0.6479				
1.0	1.8	0.0051	0.0128	0.0257	0.0521	0.0792	0.1072	0.2971	0.5508	0.6042	0.6625	0.7272	0.7624	0.7847	0.3283	0.2218	0.6756				
1.0	2.0	0.0056	0.0141	0.0285	0.0578	0.0881	0.1194	0.3364	0.6455	0.7154	0.7948	0.8875	0.9407	0.9754	0.3832	0.2697	0.7039				
1.0	2.2	0.0060	0.0151	0.0304	0.0617	0.0942	0.1278	0.3640	0.7185	0.8040	0.9052	1.0307	1.1078	1.1609	0.4266	0.3121	0.7317				
1.0	2.4	0.0062	0.0157	0.0316	0.0642	0.0981	0.1332	0.3824	0.7709	0.8696	0.9909	1.1514	1.2584	1.3377	0.4594	0.3481	0.7578				
1.0	2.6	0.0064	0.0160	0.0323	0.0658	0.1005	0.1366	0.3940	0.8060	0.9148	1.0525	1.2457	1.3862	1.5005	0.4831	0.3774	0.7811				
1.0	2.8	0.0065	0.0163	0.0328	0.0667	0.1020	0.1386	0.4010	0.8280	0.9436	1.0932	1.3130	1.4861	1.6424	0.4994	0.3999	0.8008				
1.0	3.0	0.0065	0.0164	0.0330	0.0673	0.1028	0.1398	0.4051	0.8410	0.9609	1.1182	1.3569	1.5568	1.7563	0.5101	0.4165	0.8165				
1.2	1.4	0.0018	0.0044	0.0089	0.0179	0.0269	0.0361	0.0935	0.1556	0.1664	0.1774	0.1886	0.1943	0.1977	0.0957	0.0576	0.6021				
1.2	1.6	0.0031	0.0078	0.0157	0.0316	0.0479	0.0646	0.1726	0.2996	0.3233	0.3479	0.3734	0.3866	0.3946	0.1815	0.1143	0.6296				
1.2	1.8	0.0041	0.0103	0.0206	0.0418	0.0635	0.0859	0.2360	0.4280	0.4666	0.5077	0.5520	0.5755	0.5901	0.2561	0.1688	0.6593				
1.2	2.0	0.0048	0.0120	0.0241	0.0490	0.0746	0.1011	0.2840	0.5368	0.5916	0.6524	0.7210	0.7590	0.7832	0.3186	0.2199	0.6902				
1.2	2.2	0.0052	0.0131	0.0265	0.0538	0.0822	0.1115	0.3182	0.6232	0.6945	0.7770	0.8757	0.9337	0.9723	0.3688	0.2660	0.7211				
1.2	2.4	0.0055	0.0139	0.0280	0.0570	0.0870	0.1183	0.3412	0.6871	0.7733	0.8773	1.0103	1.0949	1.1548	0.4074	0.3059	0.7508				
1.2	2.6	0.0057	0.0143	0.0289	0.0589	0.0901	0.1226	0.3559	0.7310	0.8291	0.9519	1.1196	1.2362	1.3262	0.4358	0.3390	0.7780				
1.2	2.8	0.0058	0.0146	0.0295	0.0601	0.0919	0.1251	0.3648	0.7590	0.8656	1.0027	1.2007	1.3512	1.4801	0.4555	0.3651	0.8015				
1.2	3.0	0.0059	0.0148	0.0298	0.0608	0.0930	0.1266	0.3700	0.7757	0.8878	1.0346	1.2554	1.4361	1.6087	0.4686	0.3846	0.8207				
1.4	1.6	0.0017	0.0043	0.0087	0.0175	0.0265	0.0355	0.0925	0.1549	0.1658	0.1770	0.1884	0.1942	0.1977	0.0950	0.0576	0.6065				
1.4	1.8	0.0030	0.0075	0.0151	0.0306	0.0464	0.0626	0.1688	0.2965	0.3207	0.3459	0.3723	0.3860	0.3944	0.1789	0.1140	0.6374				
1.4	2.0	0.0039	0.0098	0.0196	0.0398	0.0607	0.0821	0.2281	0.4204	0.4599	0.5025	0.5490	0.5738	0.5894	0.2504	0.1679	0.6705				
1.4	2.2	0.0045	0.0112	0.0227	0.0461	0.0704	0.0955	0.2713	0.4786	0.5225	0.5786	0.6417	0.7143	0.7815	0.3090	0.2177	0.7045				
1.4	2.4	0.0049	0.0122	0.0247	0.0502	0.0767	0.1044	0.3009	0.6008	0.6731	0.7583	0.8627	0.9259	0.9688	0.3549	0.2619	0.7379				
1.4	2.6	0.0051	0.0128	0.0259	0.0527	0.0807	0.1099	0.3201	0.6562	0.7425	0.8485	0.9882	1.0803	1.1477	0.3891	0.2993	0.7693				
1.4	2.8	0.0052	0.0132	0.0266	0.0543	0.0831	0.1132	0.3318	0.6925	0.7894	0.9125	1.0857	1.2113	1.3127	0.4134	0.3295	0.7971				
1.4	3.0	0.0053	0.0134	0.0270	0.0551	0.0844	0.1151	0.3386	0.7146	0.8185	0.9538	1.1543	1.3129	1.4562	0.4298	0.3526	0.8205				

Table 1. (continued)

k_a	k_B	Cumulative Probability, $F(t)$																Mean	Std Dev	COV
		0.01	0.025	0.05	0.1	0.15	0.2	0.5	0.8	0.85	0.9	0.95	0.975	0.99						
1.6	1.8	0.0017	0.0043	0.0086	0.0172	0.0261	0.0350	0.0916	0.1541	0.1653	0.1766	0.1882	0.1941	0.1976	0.0943	0.0576	0.6107			
1.6	2.0	0.0029	0.0073	0.0146	0.0296	0.0449	0.0607	0.1651	0.2933	0.3180	0.3439	0.3712	0.3854	0.3941	0.1763	0.1137	0.6451			
1.6	2.2	0.0037	0.0093	0.0187	0.0380	0.0579	0.0785	0.2204	0.4125	0.4530	0.4971	0.5457	0.5720	0.5886	0.2448	0.1669	0.6817			
1.6	2.4	0.0042	0.0106	0.0214	0.0435	0.0665	0.0904	0.2592	0.5079	0.5652	0.6305	0.7070	0.7510	0.7797	0.2996	0.2153	0.7186			
1.6	2.6	0.0045	0.0114	0.0230	0.0469	0.0718	0.0978	0.2848	0.5784	0.6513	0.7387	0.8487	0.9172	0.9647	0.3414	0.2575	0.7542			
1.6	2.8	0.0047	0.0119	0.0240	0.0490	0.0750	0.1023	0.3006	0.6261	0.7119	0.8191	0.9644	1.0640	1.1394	0.3716	0.2924	0.7868			
1.6	3.0	0.0048	0.0122	0.0246	0.0502	0.0769	0.1049	0.3099	0.6559	0.7508	0.8732	1.0502	1.1838	1.2969	0.3923	0.3198	0.8150			
1.8	2.0	0.0017	0.0042	0.0084	0.0169	0.0256	0.0344	0.0906	0.1534	0.1647	0.1762	0.1879	0.1939	0.1976	0.0937	0.0575	0.6142			
1.8	2.2	0.0028	0.0070	0.0141	0.0286	0.0435	0.0588	0.1614	0.2900	0.3152	0.3418	0.3700	0.3847	0.3938	0.1738	0.1134	0.6524			
1.8	2.4	0.0035	0.0089	0.0178	0.0363	0.0554	0.0752	0.2129	0.4045	0.4458	0.4914	0.5422	0.5701	0.5878	0.2393	0.1657	0.6925			
1.8	2.6	0.0040	0.0100	0.0202	0.0411	0.0629	0.0856	0.2477	0.4933	0.5514	0.6187	0.6991	0.7463	0.7776	0.2905	0.2127	0.7324			
1.8	2.8	0.0042	0.0107	0.0215	0.0440	0.0674	0.0919	0.2696	0.5562	0.6294	0.7185	0.8336	0.9075	0.9601	0.3284	0.2528	0.7700			
1.8	3.0	0.0044	0.0111	0.0223	0.0456	0.0700	0.0955	0.2827	0.5970	0.6817	0.7893	0.9393	1.0459	1.1298	0.3549	0.2852	0.8035			
2.0	2.2	0.0016	0.0041	0.0083	0.0167	0.0252	0.0339	0.0896	0.1527	0.1640	0.1757	0.1877	0.1938	0.1975	0.0930	0.0573	0.6162			
2.0	2.4	0.0027	0.0068	0.0137	0.0277	0.0422	0.0571	0.1577	0.2867	0.3123	0.3396	0.3687	0.3841	0.3936	0.1712	0.1129	0.6594			
2.0	2.6	0.0034	0.0085	0.0170	0.0347	0.0530	0.0720	0.2056	0.3964	0.4385	0.4854	0.5385	0.5681	0.5869	0.2339	0.1644	0.7031			
2.0	2.8	0.0038	0.0094	0.0191	0.0389	0.0595	0.0811	0.2367	0.4786	0.5374	0.6064	0.6907	0.7413	0.7754	0.2815	0.2099	0.7457			
2.0	3.0	0.0040	0.0100	0.0202	0.0413	0.0633	0.0864	0.2555	0.5345	0.6075	0.6978	0.8175	0.8968	0.9549	0.3158	0.2480	0.7851			
2.2	2.4	0.0016	0.0040	0.0081	0.0164	0.0248	0.0334	0.0886	0.1519	0.1634	0.1753	0.1874	0.1937	0.1975	0.0924	0.0570	0.6169			
2.2	2.6	0.0026	0.0066	0.0132	0.0268	0.0409	0.0554	0.1542	0.2832	0.3094	0.3373	0.3674	0.3833	0.3933	0.1687	0.1123	0.6660			
2.2	2.8	0.0032	0.0081	0.0163	0.0332	0.0507	0.0690	0.1986	0.3881	0.4309	0.4792	0.5346	0.5658	0.5859	0.2285	0.1630	0.7135			
2.2	3.0	0.0035	0.0089	0.0180	0.0369	0.0565	0.0770	0.2263	0.4640	0.5232	0.5937	0.6817	0.7357	0.7728	0.2728	0.2070	0.7587			
2.4	2.6	0.0016	0.0040	0.0080	0.0161	0.0244	0.0329	0.0877	0.1511	0.1628	0.1748	0.1872	0.1935	0.1974	0.0917	0.0566	0.6171			
2.4	2.8	0.0025	0.0063	0.0128	0.0260	0.0396	0.0537	0.1506	0.2798	0.3063	0.3350	0.3660	0.3826	0.3929	0.1661	0.1118	0.6727			
2.4	3.0	0.0031	0.0077	0.0156	0.0318	0.0486	0.0662	0.1918	0.3798	0.4232	0.4727	0.5305	0.5634	0.5849	0.2232	0.1616	0.7238			
2.6	2.8	0.0016	0.0039	0.0078	0.0158	0.0240	0.0323	0.0867	0.1504	0.1622	0.1743	0.1869	0.1934	0.1973	0.0911	0.0564	0.6188			
2.6	3.0	0.0024	0.0061	0.0124	0.0252	0.0384	0.0522	0.1472	0.2763	0.3032	0.3325	0.3646	0.3818	0.3926	0.1636	0.1113	0.6801			
2.8	3.0	0.0015	0.0038	0.0077	0.0156	0.0236	0.0318	0.0857	0.1496	0.1615	0.1739	0.1867	0.1933	0.1973	0.0904	0.0566	0.6261			

Table 2. Mean, Standard Deviation, and Coefficient of Variation Associated with a Standardized, Doubly-Truncated Normal Distribution

k_L	k_R	Mean	Std Dev	CoV	k_L	k_R	Mean	Std Dev	CoV
-3.0	-2.8	0.1096	0.0566	0.5166	-2.6	-2.4	0.1083	0.0566	0.5229
-3.0	-2.6	0.2364	0.1113	0.4709	-2.6	-2.2	0.2313	0.1123	0.4856
-3.0	-2.4	0.3768	0.1616	0.4288	-2.6	-2.0	0.3661	0.1644	0.4491
-3.0	-2.2	0.5272	0.2070	0.3927	-2.6	-1.8	0.5095	0.2127	0.4175
-3.0	-2.0	0.6842	0.2480	0.3624	-2.6	-1.6	0.6586	0.2575	0.3910
-3.0	-1.8	0.8451	0.2852	0.3375	-2.6	-1.4	0.8109	0.2993	0.3692
-3.0	-1.6	1.0077	0.3198	0.3173	-2.6	-1.2	0.9642	0.3390	0.3516
-3.0	-1.4	1.1702	0.3526	0.3013	-2.6	-1.0	1.1169	0.3774	0.3379
-3.0	-1.2	1.3314	0.3846	0.2889	-2.6	-0.8	1.2674	0.4150	0.3275
-3.0	-1.0	1.4899	0.4165	0.2795	-2.6	-0.6	1.4143	0.4526	0.3200
-3.0	-0.8	1.6449	0.4488	0.2729	-2.6	-0.4	1.5565	0.4906	0.3152
-3.0	-0.6	1.7952	0.4820	0.2685	-2.6	-0.2	1.6928	0.5293	0.3126
-3.0	-0.4	1.9400	0.5165	0.2662	-2.6	0.0	1.8220	0.5687	0.3121
-3.0	-0.2	2.0782	0.5523	0.2657	-2.6	0.2	1.9431	0.6090	0.3134
-3.0	0.0	2.2088	0.5894	0.2668	-2.6	0.4	2.0550	0.6499	0.3162
-3.0	0.2	2.3310	0.6278	0.2693	-2.6	0.6	2.1567	0.6908	0.3203
-3.0	0.4	2.4437	0.6671	0.2730	-2.6	0.8	2.2476	0.7314	0.3254
-3.0	0.6	2.5461	0.7068	0.2776	-2.6	1.0	2.3270	0.7708	0.3312
-3.0	0.8	2.6374	0.7464	0.2830	-2.6	1.2	2.3948	0.8081	0.3374
-3.0	1.0	2.7172	0.7849	0.2889	-2.6	1.4	2.4511	0.8426	0.3438
-3.0	1.2	2.7852	0.8216	0.2950	-2.6	1.6	2.4965	0.8735	0.3499
-3.0	1.4	2.8417	0.8556	0.3011	-2.6	1.8	2.5319	0.9002	0.3556
-3.0	1.6	2.8872	0.8861	0.3069	-2.6	2.0	2.5585	0.9225	0.3606
-3.0	1.8	2.9226	0.9125	0.3122	-2.6	2.2	2.5777	0.9402	0.3647
-3.0	2.0	2.9492	0.9344	0.3168	-2.6	2.4	2.5911	0.9538	0.3681
-3.0	2.2	2.9685	0.9520	0.3207	-2.6	2.6	2.6000	0.9637	0.3707
-3.0	2.4	2.9819	0.9654	0.3238	-2.6	2.8	2.6057	0.9706	0.3725
-3.0	2.6	2.9908	0.9752	0.3261	-2.6	3.0	2.6092	0.9752	0.3738
-3.0	2.8	2.9965	0.9820	0.3277	-2.4	-2.2	0.1076	0.0570	0.5296
-3.0	3.0	3.0000	0.9866	0.3289	-2.4	-2.0	0.2288	0.1129	0.4934
-2.8	-2.6	0.1089	0.0564	0.5175	-2.4	-1.8	0.3607	0.1657	0.4595
-2.8	-2.4	0.2339	0.1118	0.4779	-2.4	-1.6	0.5004	0.2153	0.4303
-2.8	-2.2	0.3715	0.1630	0.4389	-2.4	-1.4	0.6451	0.2619	0.4059
-2.8	-2.0	0.5185	0.2099	0.4050	-2.4	-1.2	0.7926	0.3059	0.3860
-2.8	-1.8	0.6716	0.2528	0.3765	-2.4	-1.0	0.9406	0.3481	0.3701
-2.8	-1.6	0.8284	0.2924	0.3530	-2.4	-0.8	1.0875	0.3892	0.3578
-2.8	-1.4	0.9866	0.3295	0.3340	-2.4	-0.6	1.2317	0.4296	0.3488
-2.8	-1.2	1.1445	0.3651	0.3190	-2.4	-0.4	1.3718	0.4700	0.3426
-2.8	-1.0	1.3006	0.3999	0.3075	-2.4	-0.2	1.5064	0.5107	0.3390
-2.8	-0.8	1.4537	0.4347	0.2990	-2.4	0.0	1.6344	0.5518	0.3376
-2.8	-0.6	1.6027	0.4698	0.2931	-2.4	0.2	1.7545	0.5935	0.3383
-2.8	-0.4	1.7464	0.5058	0.2896	-2.4	0.4	1.8656	0.6354	0.3406
-2.8	-0.2	1.8838	0.5428	0.2881	-2.4	0.6	1.9668	0.6774	0.3444
-2.8	0.0	2.0139	0.5810	0.2885	-2.4	0.8	2.0573	0.7187	0.3493
-2.8	0.2	2.1357	0.6202	0.2904	-2.4	1.0	2.1364	0.7587	0.3551
-2.8	0.4	2.2480	0.6601	0.2936	-2.4	1.2	2.2041	0.7966	0.3614
-2.8	0.6	2.3502	0.7004	0.2980	-2.4	1.4	2.2602	0.8315	0.3679
-2.8	0.8	2.4413	0.7404	0.3033	-2.4	1.6	2.3055	0.8627	0.3742
-2.8	1.0	2.5210	0.7793	0.3091	-2.4	1.8	2.3408	0.8897	0.3801
-2.8	1.2	2.5889	0.8162	0.3153	-2.4	2.0	2.3674	0.9122	0.3853
-2.8	1.4	2.6453	0.8504	0.3215	-2.4	2.2	2.3866	0.9301	0.3897
-2.8	1.6	2.6907	0.8811	0.3274	-2.4	2.4	2.4000	0.9438	0.3932
-2.8	1.8	2.7261	0.9076	0.3329	-2.4	2.6	2.4089	0.9538	0.3959
-2.8	2.0	2.7527	0.9296	0.3377	-2.4	2.8	2.4146	0.9608	0.3979
-2.8	2.2	2.7720	0.9473	0.3417	-2.4	3.0	2.4181	0.9654	0.3992
-2.8	2.4	2.7854	0.9608	0.3449					
-2.8	2.6	2.7943	0.9706	0.3474					
-2.8	2.8	2.8000	0.9775	0.3491					
-2.8	3.0	2.8035	0.9820	0.3503					

Table 2. (continued)

k_L	k_R	Mean	Std Dev	CoV	k_L	k_R	Mean	Std Dev	CoV
-2.2	-2.0	0.1070	0.0573	0.5360	-1.8	-1.6	0.1057	0.0576	0.5454
-2.2	-1.8	0.2262	0.1134	0.5011	-1.8	-1.4	0.2211	0.1140	0.5158
-2.2	-1.6	0.3552	0.1669	0.4699	-1.8	-1.2	0.3439	0.1688	0.4910
-2.2	-1.4	0.4910	0.2177	0.4434	-1.8	-1.0	0.4717	0.2218	0.4703
-2.2	-1.2	0.6312	0.2660	0.4214	-1.8	-0.8	0.6021	0.2731	0.4536
-2.2	-1.0	0.7734	0.3121	0.4035	-1.8	-0.6	0.7331	0.3229	0.4405
-2.2	-0.8	0.9158	0.3567	0.3895	-1.8	-0.4	0.8626	0.3717	0.4309
-2.2	-0.6	1.0563	0.4003	0.3789	-1.8	-0.2	0.9890	0.4197	0.4244
-2.2	-0.4	1.1936	0.4434	0.3715	-1.8	0.0	1.1105	0.4672	0.4208
-2.2	-0.2	1.3260	0.4864	0.3668	-1.8	0.2	1.2256	0.5144	0.4197
-2.2	0.0	1.4523	0.5295	0.3646	-1.8	0.4	1.3330	0.5610	0.4208
-2.2	0.2	1.5711	0.5728	0.3646	-1.8	0.6	1.4314	0.6069	0.4240
-2.2	0.4	1.6812	0.6161	0.3665	-1.8	0.8	1.5198	0.6515	0.4287
-2.2	0.6	1.7817	0.6592	0.3700	-1.8	1.0	1.5976	0.6944	0.4346
-2.2	0.8	1.8717	0.7015	0.3748	-1.8	1.2	1.6643	0.7347	0.4414
-2.2	1.0	1.9504	0.7423	0.3806	-1.8	1.4	1.7199	0.7716	0.4486
-2.2	1.2	2.0178	0.7808	0.3870	-1.8	1.6	1.7648	0.8045	0.4559
-2.2	1.4	2.0738	0.8163	0.3936	-1.8	1.8	1.8000	0.8329	0.4627
-2.2	1.6	2.1190	0.8480	0.4002	-1.8	2.0	1.8265	0.8565	0.4689
-2.2	1.8	2.1542	0.8753	0.4063	-1.8	2.2	1.8458	0.8753	0.4742
-2.2	2.0	2.1808	0.8981	0.4118	-1.8	2.4	1.8592	0.8897	0.4786
-2.2	2.2	2.2000	0.9162	0.4165	-1.8	2.6	1.8681	0.9002	0.4819
-2.2	2.4	2.2134	0.9301	0.4202	-1.8	2.8	1.8739	0.9076	0.4843
-2.2	2.6	2.2223	0.9402	0.4231	-1.8	3.0	1.8774	0.9125	0.4860
-2.2	2.8	2.2280	0.9473	0.4252	-1.6	-1.4	0.1050	0.0576	0.5489
-2.2	3.0	2.2315	0.9520	0.4266	-1.6	-1.2	0.2185	0.1143	0.5231
-2.0	-1.8	0.1063	0.0575	0.5412	-1.6	-1.0	0.3382	0.1696	0.5017
-2.0	-1.6	0.2237	0.1137	0.5085	-1.6	-0.8	0.4617	0.2235	0.4841
-2.0	-1.4	0.3496	0.1679	0.4804	-1.6	-0.6	0.5870	0.2761	0.4703
-2.0	-1.2	0.4814	0.2199	0.4567	-1.6	-0.4	0.7119	0.3274	0.4599
-2.0	-1.0	0.6168	0.2697	0.4373	-1.6	-0.2	0.8345	0.3779	0.4528
-2.0	-0.8	0.7536	0.3178	0.4217	-1.6	0.0	0.9531	0.4276	0.4486
-2.0	-0.6	0.8897	0.3646	0.4098	-1.6	0.2	1.0659	0.4767	0.4472
-2.0	-0.4	1.0235	0.4106	0.4012	-1.6	0.4	1.1715	0.5250	0.4482
-2.0	-0.2	1.1531	0.4560	0.3955	-1.6	0.6	1.2687	0.5725	0.4512
-2.0	0.0	1.2772	0.5013	0.3925	-1.6	0.8	1.3562	0.6185	0.4560
-2.0	0.2	1.3943	0.5465	0.3919	-1.6	1.0	1.4334	0.6625	0.4622
-2.0	0.4	1.5033	0.5914	0.3934	-1.6	1.2	1.4997	0.7039	0.4693
-2.0	0.6	1.6028	0.6358	0.3967	-1.6	1.4	1.5551	0.7417	0.4770
-2.0	0.8	1.6921	0.6792	0.4014	-1.6	1.6	1.6000	0.7755	0.4847
-2.0	1.0	1.7704	0.7209	0.4072	-1.6	1.8	1.6352	0.8045	0.4920
-2.0	1.2	1.8374	0.7603	0.4138	-1.6	2.0	1.6617	0.8287	0.4987
-2.0	1.4	1.8932	0.7964	0.4207	-1.6	2.2	1.6810	0.8480	0.5044
-2.0	1.6	1.9383	0.8287	0.4275	-1.6	2.4	1.6945	0.8627	0.5091
-2.0	1.8	1.9735	0.8565	0.4340	-1.6	2.6	1.7035	0.8735	0.5128
-2.0	2.0	2.0000	0.8796	0.4398	-1.6	2.8	1.7093	0.8811	0.5155
-2.0	2.2	2.0192	0.8981	0.4448	-1.6	3.0	1.7128	0.8861	0.5173
-2.0	2.4	2.0326	0.9122	0.4488					
-2.0	2.6	2.0415	0.9225	0.4518					
-2.0	2.8	2.0473	0.9296	0.4541					
-2.0	3.0	2.0508	0.9344	0.4556					

Table 2. (continued)

k_L	k_R	Mean	Std Dev	CoV	k_L	k_R	Mean	Std Dev	CoV
-1.4	-1.2	0.1043	0.0576	0.5522	-1.0	-0.8	0.1030	0.0576	0.5595
-1.4	-1.0	0.2159	0.1145	0.5305	-1.0	-0.6	0.2106	0.1149	0.5455
-1.4	-0.8	0.3324	0.1703	0.5125	-1.0	-0.4	0.3207	0.1714	0.5346
-1.4	-0.6	0.4517	0.2250	0.4982	-1.0	-0.2	0.4312	0.2272	0.5270
-1.4	-0.4	0.5716	0.2786	0.4874	-1.0	0.0	0.5401	0.2822	0.5225
-1.4	-0.2	0.6902	0.3312	0.4798	-1.0	0.2	0.6456	0.3364	0.5210
-1.4	0.0	0.8056	0.3829	0.4754	-1.0	0.4	0.7458	0.3895	0.5223
-1.4	0.2	0.9159	0.4339	0.4737	-1.0	0.6	0.8391	0.4414	0.5261
-1.4	0.4	1.0197	0.4840	0.4747	-1.0	0.8	0.9242	0.4916	0.5320
-1.4	0.6	1.1155	0.5330	0.4778	-1.0	1.0	1.0000	0.5396	0.5396
-1.4	0.8	1.2021	0.5804	0.4828	-1.0	1.2	1.0658	0.5845	0.5484
-1.4	1.0	1.2787	0.6257	0.4894	-1.0	1.4	1.1213	0.6257	0.5581
-1.4	1.2	1.3447	0.6682	0.4969	-1.0	1.6	1.1666	0.6625	0.5679
-1.4	1.4	1.4000	0.7071	0.5051	-1.0	1.8	1.2024	0.6944	0.5775
-1.4	1.6	1.4449	0.7417	0.5134	-1.0	2.0	1.2296	0.7209	0.5863
-1.4	1.8	1.4801	0.7716	0.5213	-1.0	2.2	1.2496	0.7423	0.5940
-1.4	2.0	1.5068	0.7964	0.5286	-1.0	2.4	1.2636	0.7587	0.6004
-1.4	2.2	1.5262	0.8163	0.5349	-1.0	2.6	1.2730	0.7708	0.6055
-1.4	2.4	1.5398	0.8315	0.5400	-1.0	2.8	1.2790	0.7793	0.6093
-1.4	2.6	1.5489	0.8426	0.5440	-1.0	3.0	1.2828	0.7849	0.6119
-1.4	2.8	1.5547	0.8504	0.5470	-0.8	-0.6	0.1023	0.0577	0.5636
-1.4	3.0	1.5583	0.8556	0.5491	-0.8	-0.4	0.2080	0.1150	0.5530
-1.2	-1.0	0.1037	0.0576	0.5557	-0.8	-0.2	0.3148	0.1718	0.5457
-1.2	-0.8	0.2132	0.1147	0.5379	-0.8	0.0	0.4208	0.2279	0.5416
-1.2	-0.6	0.3266	0.1709	0.5235	-0.8	0.2	0.5241	0.2833	0.5405
-1.2	-0.4	0.4415	0.2263	0.5125	-0.8	0.4	0.6228	0.3377	0.5422
-1.2	-0.2	0.5560	0.2807	0.5048	-0.8	0.6	0.7153	0.3909	0.5465
-1.2	0.0	0.6681	0.3342	0.5002	-0.8	0.8	0.8000	0.4424	0.5530
-1.2	0.2	0.7759	0.3869	0.4986	-0.8	1.0	0.8758	0.4916	0.5613
-1.2	0.4	0.8778	0.4386	0.4996	-0.8	1.2	0.9419	0.5379	0.5711
-1.2	0.6	0.9723	0.4891	0.5030	-0.8	1.4	0.9979	0.5804	0.5817
-1.2	0.8	1.0581	0.5379	0.5084	-0.8	1.6	1.0438	0.6185	0.5926
-1.2	1.0	1.1342	0.5845	0.5154	-0.8	1.8	1.0802	0.6515	0.6032
-1.2	1.2	1.2000	0.6282	0.5235	-0.8	2.0	1.1079	0.6792	0.6130
-1.2	1.4	1.2553	0.6682	0.5323	-0.8	2.2	1.1283	0.7015	0.6217
-1.2	1.6	1.3003	0.7039	0.5413	-0.8	2.4	1.1427	0.7187	0.6289
-1.2	1.8	1.3357	0.7347	0.5500	-0.8	2.6	1.1524	0.7314	0.6347
-1.2	2.0	1.3626	0.7603	0.5580	-0.8	2.8	1.1587	0.7404	0.6390
-1.2	2.2	1.3822	0.7808	0.5649	-0.8	3.0	1.1626	0.7464	0.6420
-1.2	2.4	1.3959	0.7966	0.5706	-0.6	-0.4	0.1017	0.0577	0.5675
-1.2	2.6	1.4052	0.8081	0.5751	-0.6	-0.2	0.2053	0.1151	0.5606
-1.2	2.8	1.4111	0.8162	0.5784	-0.6	0.0	0.3089	0.1720	0.5569
-1.2	3.0	1.4148	0.8216	0.5808	-0.6	0.2	0.4104	0.2283	0.5563
					-0.6	0.4	0.5081	0.2838	0.5586
					-0.6	0.6	0.6000	0.3381	0.5636
					-0.6	0.8	0.6847	0.3909	0.5709
					-0.6	1.0	0.7609	0.4414	0.5801
					-0.6	1.2	0.8277	0.4891	0.5909
					-0.6	1.4	0.8845	0.5330	0.6026
					-0.6	1.6	0.9313	0.5725	0.6147
					-0.6	1.8	0.9686	0.6069	0.6265
					-0.6	2.0	0.9972	0.6358	0.6376
					-0.6	2.2	1.0183	0.6592	0.6474
					-0.6	2.4	1.0332	0.6774	0.6556
					-0.6	2.6	1.0433	0.6908	0.6622
					-0.6	2.8	1.0498	0.7004	0.6672
					-0.6	3.0	1.0539	0.7068	0.6707

Table 2. (continued)

k_L	k_R	Mean	Std Dev	CoV	k_L	k_R	Mean	Std Dev	CoV
-0.4	-0.2	0.1010	0.0577	0.5712	0.2	0.4	0.0990	0.0577	0.5827
-0.4	0.0	0.2027	0.1151	0.5682	0.2	0.6	0.1947	0.1151	0.5911
-0.4	0.2	0.3030	0.1722	0.5682	0.2	0.8	0.2852	0.1718	0.6024
-0.4	0.4	0.4000	0.2285	0.5712	0.2	1.0	0.3688	0.2272	0.6161
-0.4	0.6	0.4919	0.2838	0.5769	0.2	1.2	0.4440	0.2807	0.6321
-0.4	0.8	0.5772	0.3377	0.5851	0.2	1.4	0.5098	0.3312	0.6497
-0.4	1.0	0.6542	0.3895	0.5954	0.2	1.6	0.5655	0.3779	0.6682
-0.4	1.2	0.7222	0.4386	0.6073	0.2	1.8	0.6110	0.4197	0.6869
-0.4	1.4	0.7803	0.4840	0.6203	0.2	2.0	0.6469	0.4560	0.7050
-0.4	1.6	0.8285	0.5250	0.6338	0.2	2.2	0.6740	0.4864	0.7216
-0.4	1.8	0.8670	0.5610	0.6470	0.2	2.4	0.6936	0.5107	0.7362
-0.4	2.0	0.8967	0.5914	0.6595	0.2	2.6	0.7072	0.5293	0.7484
-0.4	2.2	0.9188	0.6161	0.6706	0.2	2.8	0.7162	0.5428	0.7580
-0.4	2.4	0.9344	0.6354	0.6800	0.2	3.0	0.7218	0.5523	0.7651
-0.4	2.6	0.9450	0.6499	0.6876	0.4	0.6	0.0983	0.0577	0.5867
-0.4	2.8	0.9520	0.6601	0.6934	0.4	0.8	0.1920	0.1150	0.5988
-0.4	3.0	0.9563	0.6671	0.6976	0.4	1.0	0.2793	0.1714	0.6138
-0.2	0.0	0.1003	0.0577	0.5750	0.4	1.2	0.3585	0.2263	0.6311
-0.2	0.2	0.2000	0.1152	0.5758	0.4	1.4	0.4284	0.2786	0.6504
-0.2	0.4	0.2970	0.1722	0.5796	0.4	1.6	0.4881	0.3274	0.6708
-0.2	0.6	0.3896	0.2283	0.5861	0.4	1.8	0.5374	0.3717	0.6917
-0.2	0.8	0.4759	0.2833	0.5953	0.4	2.0	0.5765	0.4106	0.7121
-0.2	1.0	0.5544	0.3364	0.6067	0.4	2.2	0.6064	0.4434	0.7312
-0.2	1.2	0.6241	0.3869	0.6199	0.4	2.4	0.6282	0.4700	0.7481
-0.2	1.4	0.6841	0.4339	0.6343	0.4	2.6	0.6435	0.4906	0.7624
-0.2	1.6	0.7341	0.4767	0.6493	0.4	2.8	0.6536	0.5058	0.7739
-0.2	1.8	0.7744	0.5144	0.6642	0.4	3.0	0.6600	0.5165	0.7825
-0.2	2.0	0.8057	0.5465	0.6783	0.6	0.8	0.0977	0.0577	0.5905
-0.2	2.2	0.8289	0.5728	0.6910	0.6	1.0	0.1894	0.1149	0.6065
-0.2	2.4	0.8455	0.5935	0.7019	0.6	1.2	0.2734	0.1709	0.6251
-0.2	2.6	0.8569	0.6090	0.7107	0.6	1.4	0.3483	0.2250	0.6460
-0.2	2.8	0.8643	0.6202	0.7175	0.6	1.6	0.4130	0.2761	0.6684
-0.2	3.0	0.8690	0.6278	0.7225	0.6	1.8	0.4669	0.3229	0.6916
0.0	0.2	0.0997	0.0577	0.5789	0.6	2.0	0.5103	0.3646	0.7145
0.0	0.4	0.1973	0.1151	0.5835	0.6	2.2	0.5437	0.4003	0.7363
0.0	0.6	0.2911	0.1720	0.5909	0.6	2.4	0.5683	0.4296	0.7560
0.0	0.8	0.3792	0.2279	0.6011	0.6	2.6	0.5857	0.4526	0.7728
0.0	1.0	0.4599	0.2822	0.6137	0.6	2.8	0.5973	0.4698	0.7865
0.0	1.2	0.5319	0.3342	0.6283	0.6	3.0	0.6048	0.4820	0.7970
0.0	1.4	0.5944	0.3829	0.6442	0.8	1.0	0.0970	0.0576	0.5941
0.0	1.6	0.6469	0.4276	0.6609	0.8	1.2	0.1868	0.1147	0.6141
0.0	1.8	0.6895	0.4672	0.6776	0.8	1.4	0.2676	0.1703	0.6365
0.0	2.0	0.7228	0.5013	0.6936	0.8	1.6	0.3383	0.2235	0.6609
0.0	2.2	0.7477	0.5295	0.7081	0.8	1.8	0.3979	0.2731	0.6863
0.0	2.4	0.7656	0.5518	0.7207	0.8	2.0	0.4464	0.3178	0.7119
0.0	2.6	0.7780	0.5687	0.7311	0.8	2.2	0.4842	0.3567	0.7366
0.0	2.8	0.7861	0.5810	0.7391	0.8	2.4	0.5125	0.3892	0.7594
0.0	3.0	0.7912	0.5894	0.7450	0.8	2.6	0.5326	0.4150	0.7792
					0.8	2.8	0.5463	0.4347	0.7957
					0.8	3.0	0.5551	0.4488	0.8085

Table 2. (continued)

k_L	k_R	Mean	Std Dev	CoV
1.0	1.2	0.0963	0.0576	0.5979
1.0	1.4	0.1841	0.1145	0.6218
1.0	1.6	0.2618	0.1696	0.6479
1.0	1.8	0.3283	0.2218	0.6756
1.0	2.0	0.3832	0.2697	0.7039
1.0	2.2	0.4266	0.3121	0.7317
1.0	2.4	0.4594	0.3481	0.7578
1.0	2.6	0.4831	0.3774	0.7811
1.0	2.8	0.4994	0.3999	0.8008
1.0	3.0	0.5101	0.4165	0.8165
1.2	1.4	0.0957	0.0576	0.6021
1.2	1.6	0.1815	0.1143	0.6296
1.2	1.8	0.2561	0.1688	0.6593
1.2	2.0	0.3186	0.2199	0.6902
1.2	2.2	0.3688	0.2660	0.7211
1.2	2.4	0.4074	0.3059	0.7508
1.2	2.6	0.4358	0.3390	0.7780
1.2	2.8	0.4555	0.3651	0.8015
1.2	3.0	0.4686	0.3846	0.8207
1.4	1.6	0.0950	0.0576	0.6065
1.4	1.8	0.1789	0.1140	0.6374
1.4	2.0	0.2504	0.1679	0.6705
1.4	2.2	0.3090	0.2177	0.7045
1.4	2.4	0.3549	0.2619	0.7379
1.4	2.6	0.3891	0.2993	0.7693
1.4	2.8	0.4134	0.3295	0.7971
1.4	3.0	0.4298	0.3526	0.8205
1.6	1.8	0.0943	0.0576	0.6107
1.6	2.0	0.1763	0.1137	0.6451
1.6	2.2	0.2448	0.1669	0.6817
1.6	2.4	0.2996	0.2153	0.7186
1.6	2.6	0.3414	0.2575	0.7542
1.6	2.8	0.3716	0.2924	0.7868
1.6	3.0	0.3923	0.3198	0.8150
1.8	2.0	0.0937	0.0575	0.6142
1.8	2.2	0.1738	0.1134	0.6524
1.8	2.4	0.2393	0.1657	0.6925
1.8	2.6	0.2905	0.2127	0.7324
1.8	2.8	0.3284	0.2528	0.7700
1.8	3.0	0.3549	0.2852	0.8035
2.0	2.2	0.0930	0.0573	0.6162
2.0	2.4	0.1712	0.1129	0.6594
2.0	2.6	0.2339	0.1644	0.7031
2.0	2.8	0.2815	0.2099	0.7457
2.0	3.0	0.3158	0.2480	0.7851
2.2	2.4	0.0924	0.0570	0.6169
2.2	2.6	0.1687	0.1123	0.6660
2.2	2.8	0.2285	0.1630	0.7135
2.2	3.0	0.2728	0.2070	0.7587
2.4	2.6	0.0917	0.0566	0.6171
2.4	2.8	0.1661	0.1118	0.6727
2.4	3.0	0.2232	0.1616	0.7238
2.6	2.8	0.0911	0.0564	0.6188
2.6	3.0	0.1636	0.1113	0.6801
2.8	3.0	0.0904	0.0566	0.6261

To further illustrate the behavior of the coefficient of variation c over the range of truncation points illustrated in Table 2, the area/contour plot in Figure 4 was generated. This clearly indicates that the coefficient of variation is non-unique over the range studied. However, given a value of the coefficient of variation c , one can clearly determine a potential range of combinations of lower and upper truncation points (k_L and k_R , respectively) over which that value is possible - and, thus, limit universe of possibilities.

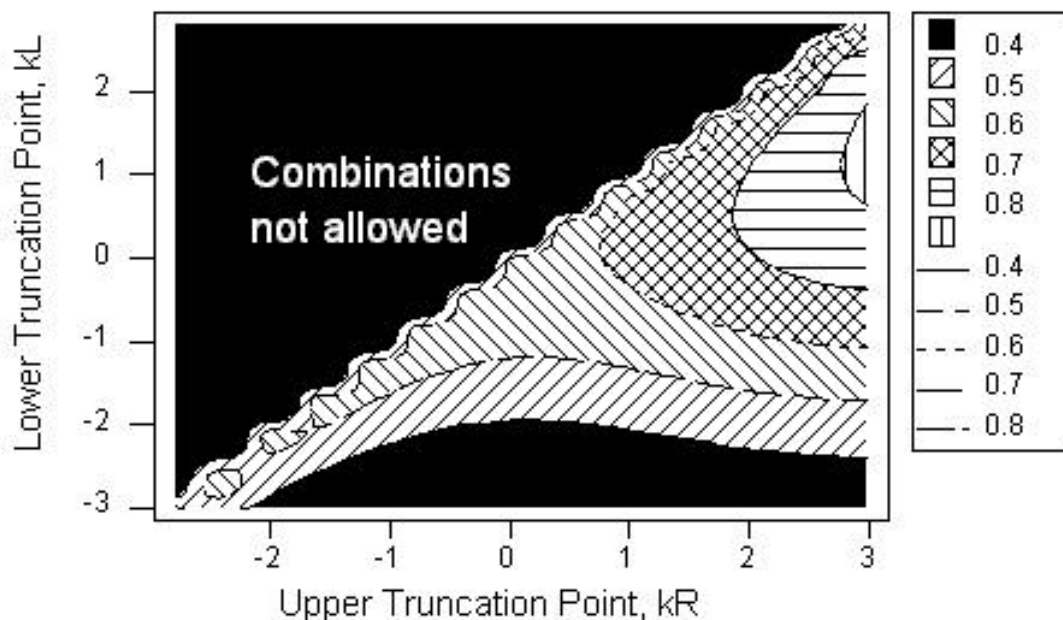


Figure 4. Contour/Area Plot of the Coefficient of Variation of the Standardized, Doubly-Truncated Normal Distribution

With respect to Figure 4, the apparent asymmetry is simply an artifact of the way in which the standardized t

value was defined - i.e., with the lower truncation point k_L set equal to 0. Had the upper truncation point k_R been set equal to 0, the resulting figure would be the mirror image of that shown in Figure 4. This "asymmetry" is eliminated when the coefficient of variation is expressed in terms of the standard normal (z) variable.

Table 3 presents the Table 1 information in a more compact form - and in a format similar to the tables often provided for functions such as the "F" distribution. In this case, for six values of the cumulative distribution function $F_{SDTN}(t)$, Table 3 provides the standardized t value associated with that $F_{SDTN}(t)$ as a function of the points of truncation k_L and k_R . Table 3 is divided into six segments - for $F_{SDTN}(t) = 0.01, 0.025, 0.05, 0.95, 0.975,$ and 0.99 .

Table 3. Standardized t Value Associated with $F(t)$ as a Function of the Points of Truncation

k_t/k_r	-3.0	-2.8	-2.6	-2.4	-2.2	-2.0	-1.8	-1.6	-1.4	-1.2	-1.0	-0.8	-0.6	-0.4	-0.2	0.0
-3.0	0.0027	0.0074	0.0151	0.0272	0.0451	0.0702	0.1031	0.1438	0.1913	0.2437	0.2988	0.3545	0.4088	0.4601	0.5072	0.5072
-2.8	0.0026	0.0071	0.0141	0.0246	0.0399	0.0606	0.0873	0.1199	0.1576	0.1990	0.2426	0.2864	0.3290	0.3691	0.4055	0.4378
-2.6	0.0025	0.0067	0.0131	0.0224	0.0353	0.0523	0.0738	0.0995	0.1288	0.1607	0.1938	0.2270	0.2589	0.2884	0.3154	0.3388
-2.4	0.0025	0.0065	0.0122	0.0203	0.0312	0.0452	0.0623	0.0824	0.1047	0.1286	0.1531	0.1772	0.2005	0.2211	0.2397	0.2555
-2.2	0.0025	0.0062	0.0114	0.0185	0.0277	0.0391	0.0527	0.0681	0.0849	0.1024	0.1200	0.1377	0.1552	0.1725	0.1896	0.2061
-2.0	0.0024	0.0059	0.0106	0.0168	0.0246	0.0339	0.0446	0.0563	0.0688	0.0815	0.0948	0.1074	0.1195	0.1311	0.1423	0.1531
-1.8	0.0024	0.0057	0.0106	0.0168	0.0246	0.0339	0.0446	0.0563	0.0688	0.0815	0.0948	0.1074	0.1195	0.1311	0.1423	0.1531
-1.6	0.0023	0.0054	0.0103	0.0164	0.0241	0.0333	0.0440	0.0557	0.0681	0.0809	0.0940	0.1074	0.1200	0.1321	0.1438	0.1552
-1.4	0.0023	0.0054	0.0103	0.0164	0.0241	0.0333	0.0440	0.0557	0.0681	0.0809	0.0940	0.1074	0.1200	0.1321	0.1438	0.1552
-1.2	0.0022	0.0050	0.0100	0.0158	0.0234	0.0326	0.0433	0.0550	0.0673	0.0800	0.0930	0.1061	0.1195	0.1321	0.1438	0.1552
-1.0	0.0022	0.0050	0.0100	0.0158	0.0234	0.0326	0.0433	0.0550	0.0673	0.0800	0.0930	0.1061	0.1195	0.1321	0.1438	0.1552
-0.8	0.0022	0.0050	0.0100	0.0158	0.0234	0.0326	0.0433	0.0550	0.0673	0.0800	0.0930	0.1061	0.1195	0.1321	0.1438	0.1552
-0.6	0.0021	0.0048	0.0096	0.0152	0.0228	0.0320	0.0427	0.0544	0.0670	0.0800	0.0926	0.1057	0.1188	0.1311	0.1423	0.1531
-0.4	0.0021	0.0048	0.0096	0.0152	0.0228	0.0320	0.0427	0.0544	0.0670	0.0800	0.0926	0.1057	0.1188	0.1311	0.1423	0.1531
-0.2	0.0020	0.0047	0.0094	0.0149	0.0224	0.0316	0.0423	0.0539	0.0664	0.0790	0.0916	0.1041	0.1166	0.1281	0.1396	0.1500
0.0	0.0020	0.0047	0.0094	0.0149	0.0224	0.0316	0.0423	0.0539	0.0664	0.0790	0.0916	0.1041	0.1166	0.1281	0.1396	0.1500
0.2	0.0019	0.0046	0.0092	0.0146	0.0220	0.0312	0.0419	0.0535	0.0659	0.0784	0.0909	0.1034	0.1159	0.1274	0.1389	0.1493
0.4	0.0019	0.0046	0.0092	0.0146	0.0220	0.0312	0.0419	0.0535	0.0659	0.0784	0.0909	0.1034	0.1159	0.1274	0.1389	0.1493
0.6	0.0019	0.0046	0.0092	0.0146	0.0220	0.0312	0.0419	0.0535	0.0659	0.0784	0.0909	0.1034	0.1159	0.1274	0.1389	0.1493
0.8	0.0018	0.0045	0.0090	0.0144	0.0218	0.0310	0.0417	0.0532	0.0655	0.0779	0.0894	0.1019	0.1144	0.1259	0.1374	0.1478
1.0	0.0018	0.0045	0.0090	0.0144	0.0218	0.0310	0.0417	0.0532	0.0655	0.0779	0.0894	0.1019	0.1144	0.1259	0.1374	0.1478
1.2	0.0018	0.0045	0.0090	0.0144	0.0218	0.0310	0.0417	0.0532	0.0655	0.0779	0.0894	0.1019	0.1144	0.1259	0.1374	0.1478
1.4	0.0017	0.0044	0.0089	0.0143	0.0217	0.0309	0.0416	0.0530	0.0652	0.0776	0.0891	0.1016	0.1141	0.1256	0.1371	0.1475
1.6	0.0017	0.0044	0.0089	0.0143	0.0217	0.0309	0.0416	0.0530	0.0652	0.0776	0.0891	0.1016	0.1141	0.1256	0.1371	0.1475
1.8	0.0017	0.0044	0.0089	0.0143	0.0217	0.0309	0.0416	0.0530	0.0652	0.0776	0.0891	0.1016	0.1141	0.1256	0.1371	0.1475
2.0	0.0017	0.0044	0.0089	0.0143	0.0217	0.0309	0.0416	0.0530	0.0652	0.0776	0.0891	0.1016	0.1141	0.1256	0.1371	0.1475
2.2	0.0017	0.0044	0.0089	0.0143	0.0217	0.0309	0.0416	0.0530	0.0652	0.0776	0.0891	0.1016	0.1141	0.1256	0.1371	0.1475
2.4	0.0016	0.0043	0.0088	0.0142	0.0216	0.0308	0.0415	0.0529	0.0650	0.0774	0.0889	0.1014	0.1139	0.1254	0.1369	0.1473
2.6	0.0016	0.0043	0.0088	0.0142	0.0216	0.0308	0.0415	0.0529	0.0650	0.0774	0.0889	0.1014	0.1139	0.1254	0.1369	0.1473
2.8	0.0016	0.0043	0.0088	0.0142	0.0216	0.0308	0.0415	0.0529	0.0650	0.0774	0.0889	0.1014	0.1139	0.1254	0.1369	0.1473
3.0	0.0015	0.0042	0.0087	0.0141	0.0215	0.0307	0.0414	0.0528	0.0649	0.0772	0.0887	0.1012	0.1137	0.1252	0.1367	0.1471

Examples of Use of the Tables for the Standardized, Doubly-Truncated Normal Distribution

Table 1. An individual is dealing with a pattern of variability (random variable) that is normally distributed with a mean value of 20 units and a standard deviation of 5 units. Due to additional constraints, the actual values of this distribution are restricted to the range from 12 units to 32 units. For planning purposes, he would like to estimate the 90th percentile of this demand pattern.

This demand pattern would be appropriately modeled as that of a doubly-truncated normal distribution. Using the notation of Figure 1: $\mu = 20$, $\sigma = 5$, $x_L = 12$, and $x_R = 32$. In terms of the standard normal distribution, the truncation points k_L and k_R can be calculated using Equation 5

$$k_L = \frac{x_L - \mu}{\sigma} = \frac{12 - 20}{5} = -1.6$$

$$k_R = \frac{x_R - \mu}{\sigma} = \frac{32 - 20}{5} = 2.4$$

Finally, to develop the standardized, doubly-truncated normal distribution for this demand pattern, use $t = z - k_L$. In this case, $t = z + 1.6$. (The upper limit of this distribution would, then, be given by $t_R = k_R - k_L = 4.0$.)

Using Table 1, with $k_L = -1.6$ and $k_R = 2.4$, one finds that the standard t value associated with $F_{SDTN}(t) = 0.90$ is $t = 2.8708$. In terms of the standard normal variate, this corresponds to

$$z = t + k_L = 2.8708 - 1.6 = 1.2706$$

This value, in turn, corresponds to a value of $x = 20 + 1.2706(5) = 26.353$ units with respect to the original distribution. This can be compared to the value of 26.4 units that would be obtained by using (non-truncated) standard normal tables.

Table 2. Continuing the example above, suppose that the manager desires to estimate the actual mean and standard deviation of the truncated population.

Using Table 2, for truncation points of $k_L = -1.6$ and $k_R = 2.4$, it can be seen that the standardized, doubly-truncated normal distribution has a mean of $\mu_t = 1.6945$ and a standard deviation of $\sigma_t = 0.8627$. In other words, the actual mean of the demand pattern will be 20.4725 units (slightly greater than the non-truncated population) with a standard deviation of 4.3135 units (slightly less than the non-truncated case).

Reference Visualizations for the Standardized, Doubly-Truncated Normal Distribution

In this section, visualizations of the standardized, doubly-truncated normal distribution are presented for reference in working with this family of distributions. Specifically - for allowable combinations of lower and upper truncations points of -3, -2, -1, 0, 1, 2, and 3 - the figures graphically represent the probability density function $f_{SDTN}(t)$ and the cumulative probability distribution $F_{SDTN}(t)$ for the appropriate range of standard t values. In addition, the mean, standard deviation, and coefficient of variation are summarized for each of these figures - based upon the values determined in Table 2.

In Figures 5 through 25, the dashed/broken line is that of the probability density function $f_{SDTN}(t)$ and the solid line is that of the cumulative distribution function $F_{SDTN}(t)$.

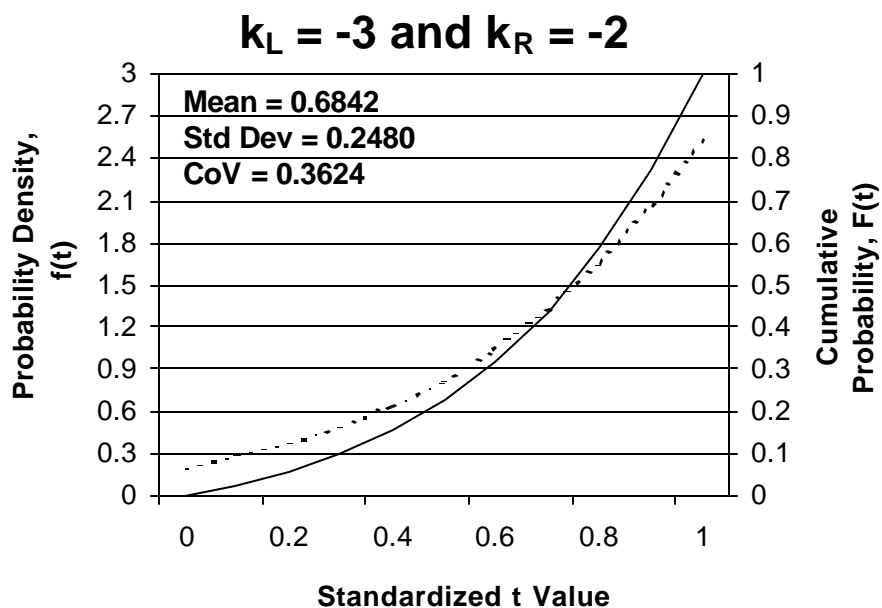


Figure 5. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = -3$ and $k_R = -2$

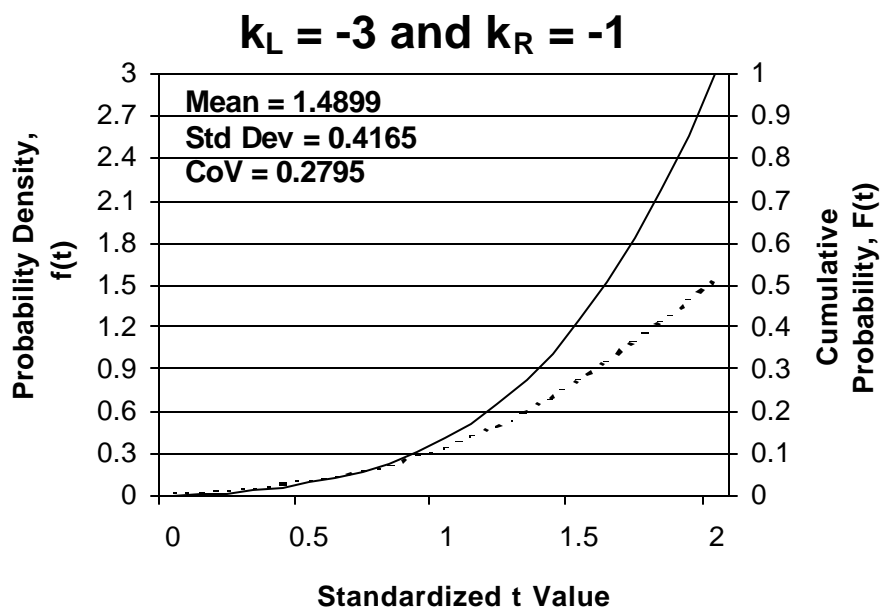


Figure 6. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = -3$ and $k_R = -1$

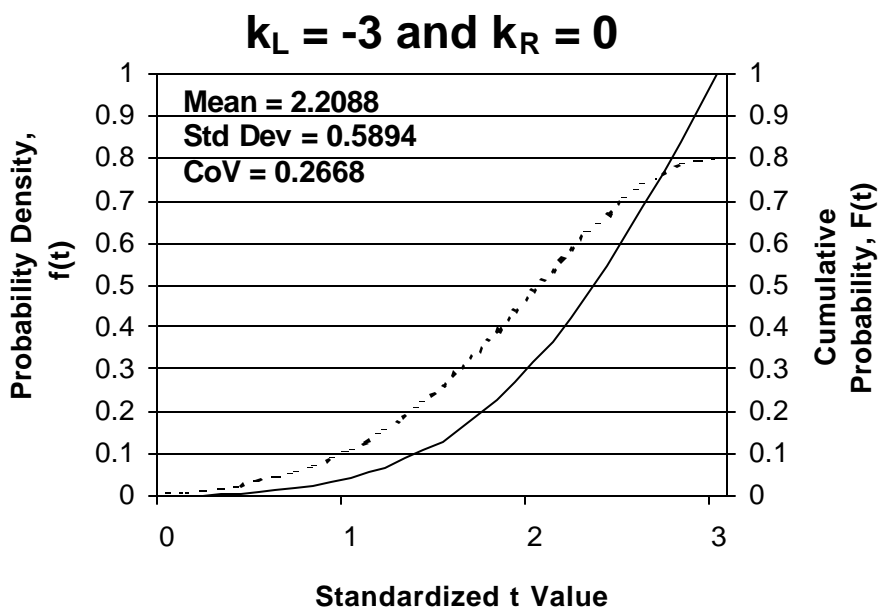


Figure 7. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = -3$ and $k_R = 0$

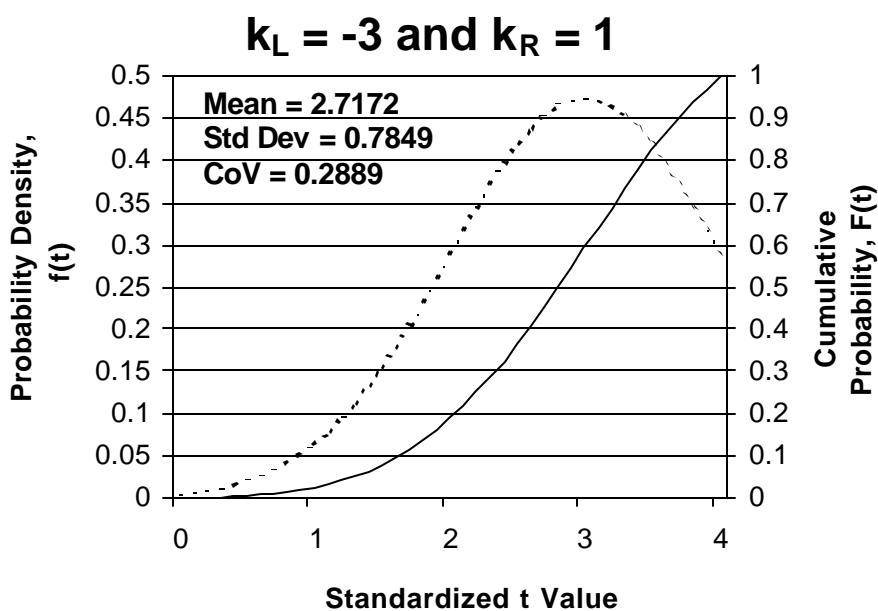


Figure 8. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = -3$ and $k_R = 1$

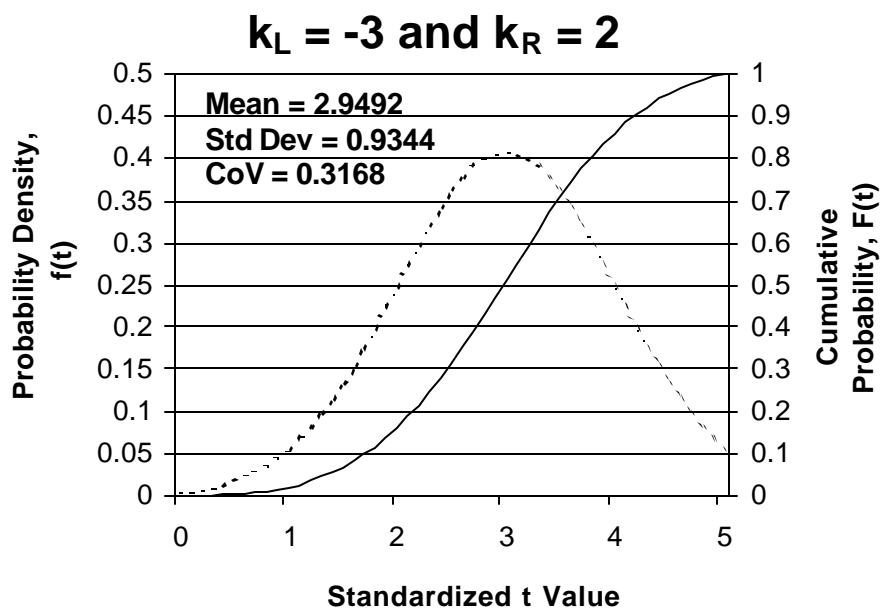


Figure 9. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = -3$ and $k_R = 2$

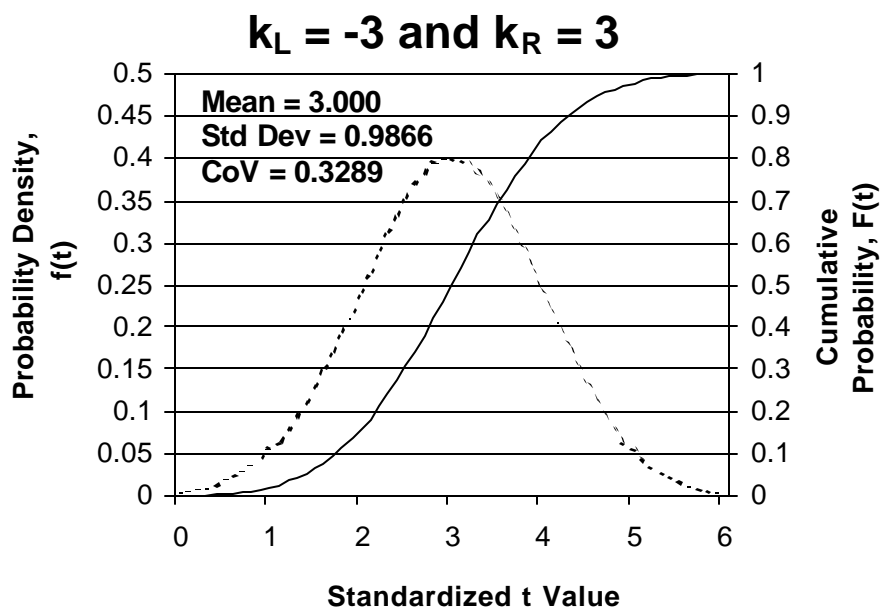


Figure 10. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = -3$ and $k_R = 3$

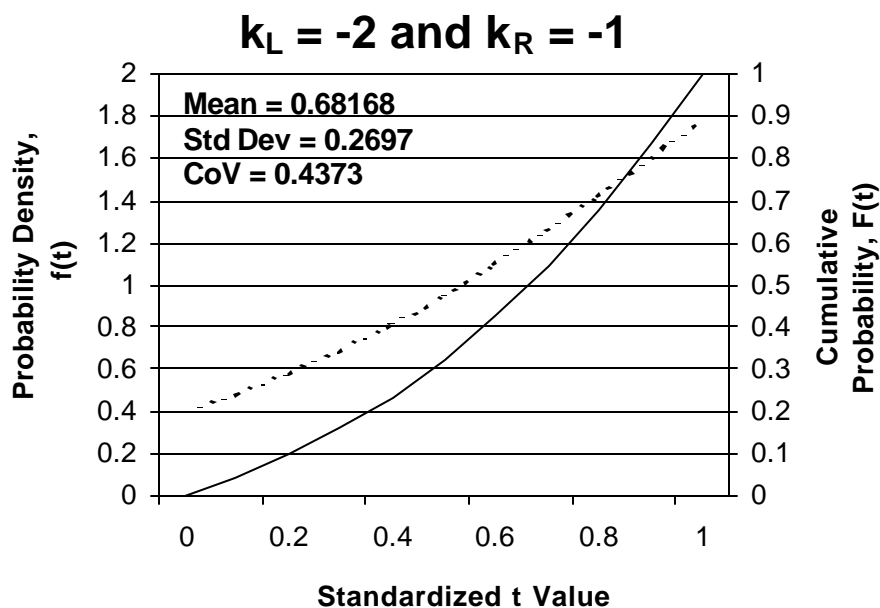


Figure 11. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = -2$ and $k_R = -1$

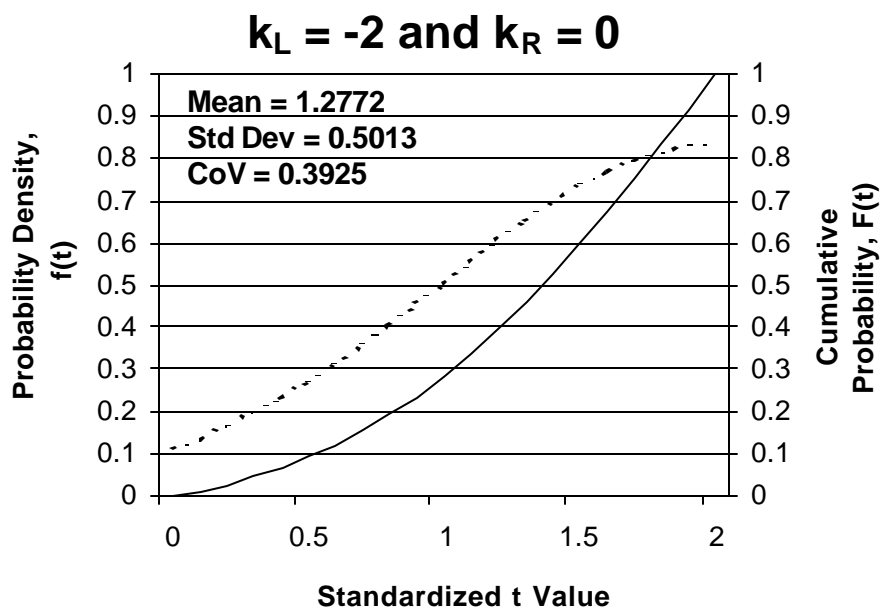


Figure 12. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = -2$ and $k_R = 0$

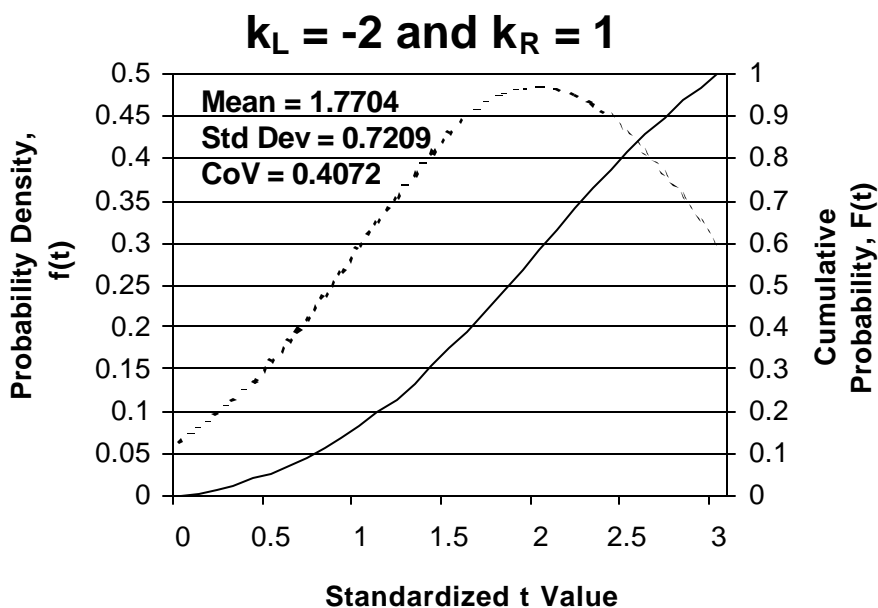


Figure 13. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = -2$ and $k_R = 1$

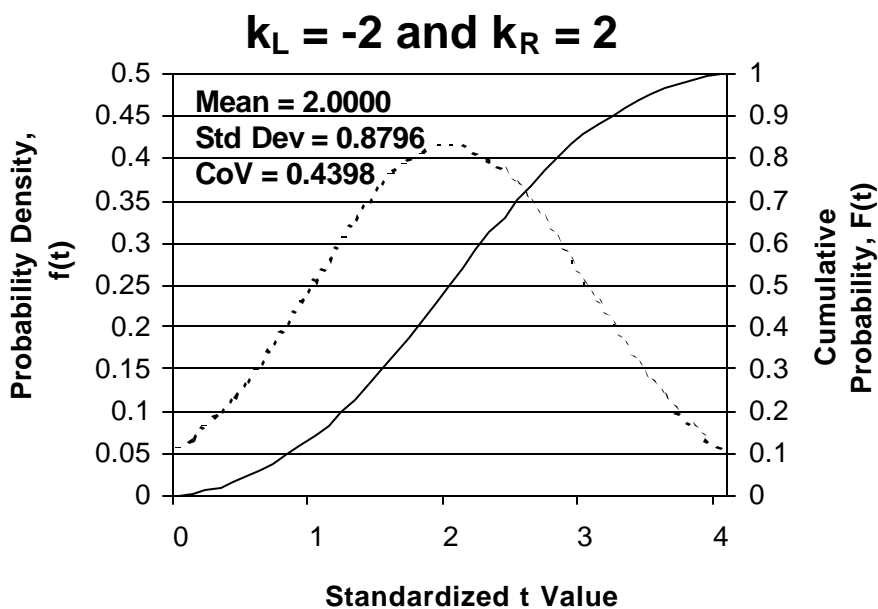


Figure 14. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = -2$ and $k_R = 2$

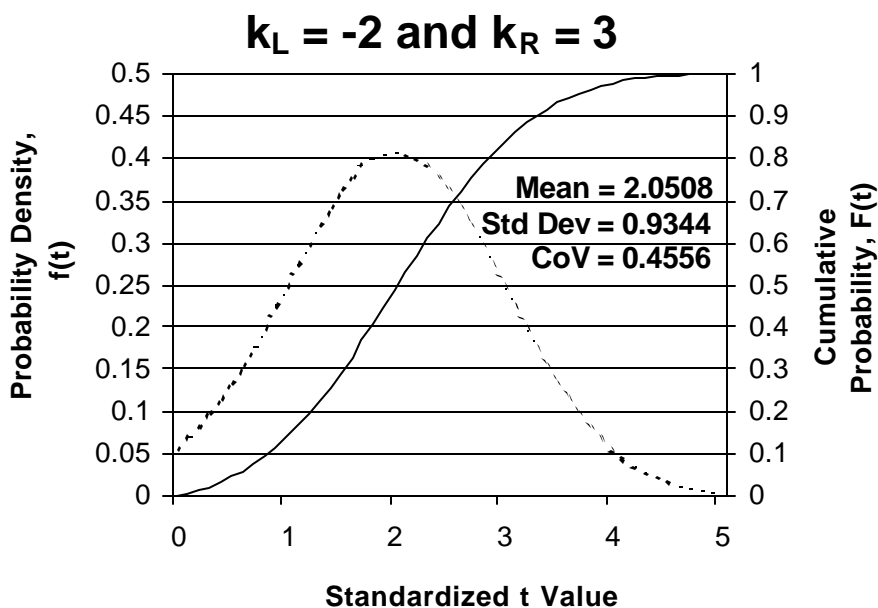


Figure 15. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = -2$ and $k_R = 3$

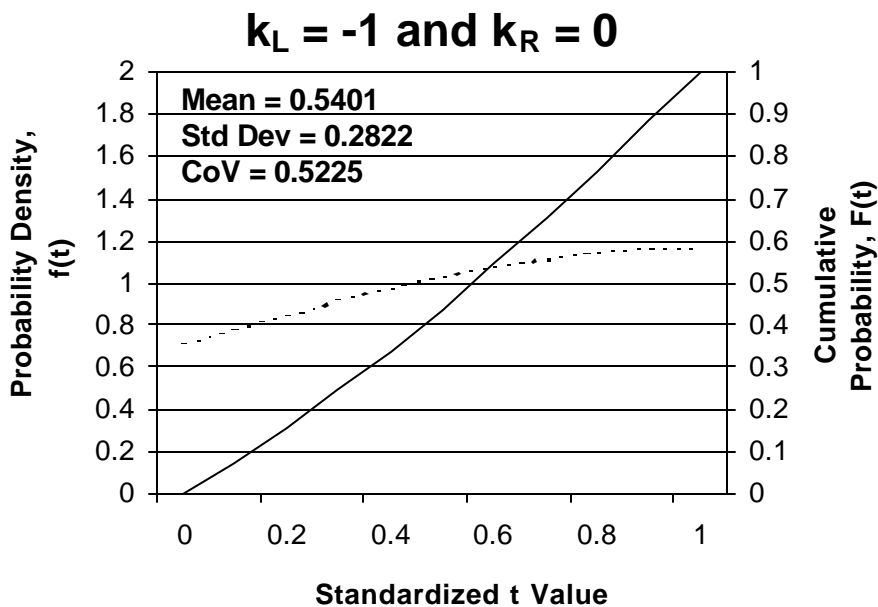


Figure 16. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = -1$ and $k_R = 0$

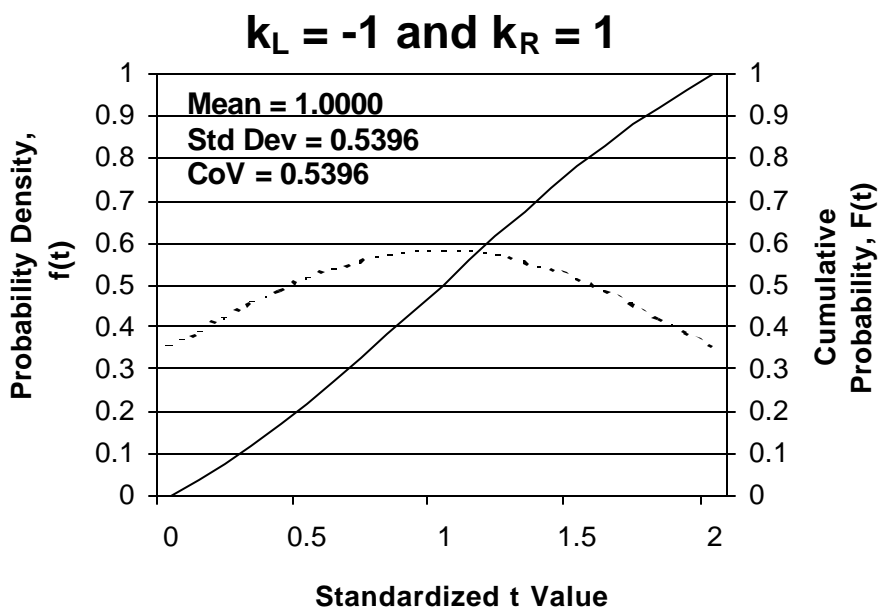


Figure 17. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = -1$ and $k_R = 1$

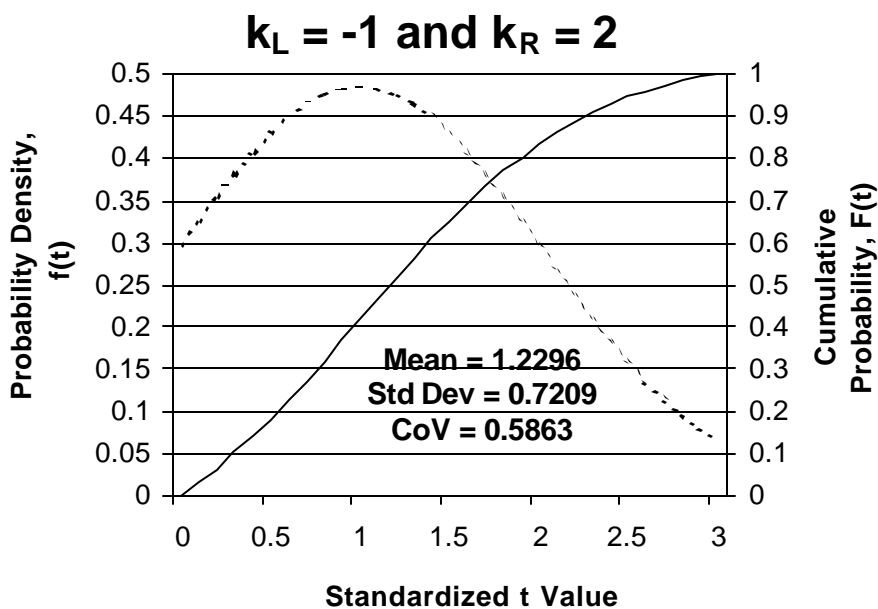


Figure 18. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = -1$ and $k_R = 2$

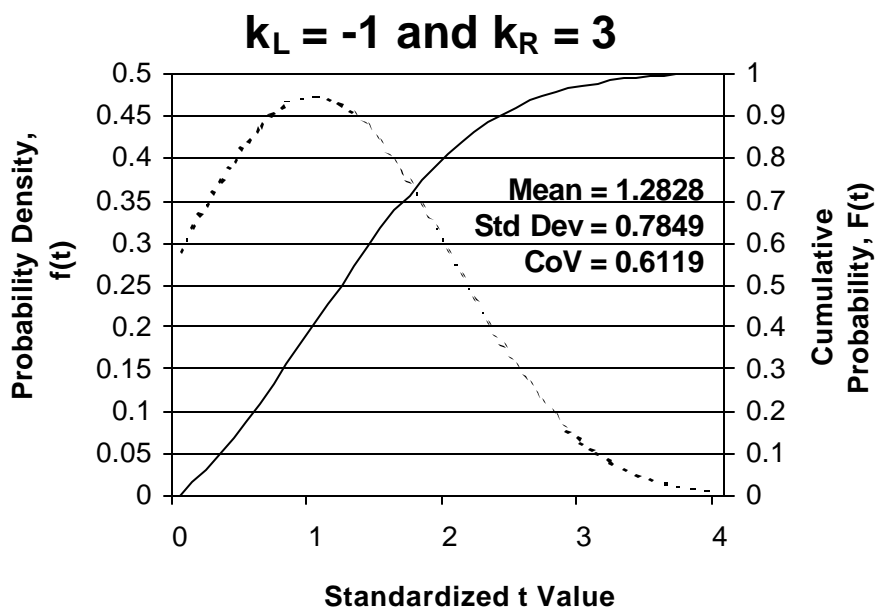


Figure 19. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = -1$ and $k_R = 3$

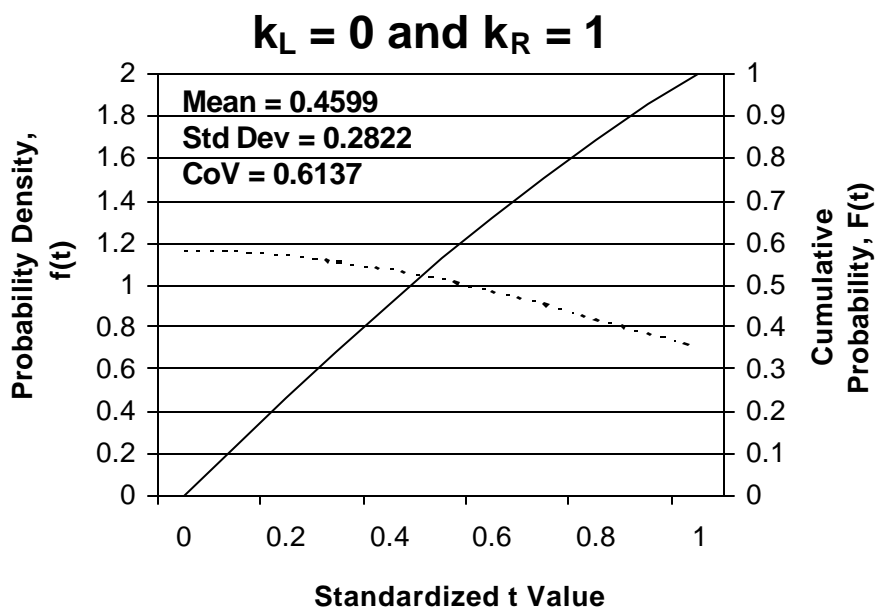


Figure 20. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = 0$ and $k_R = 1$

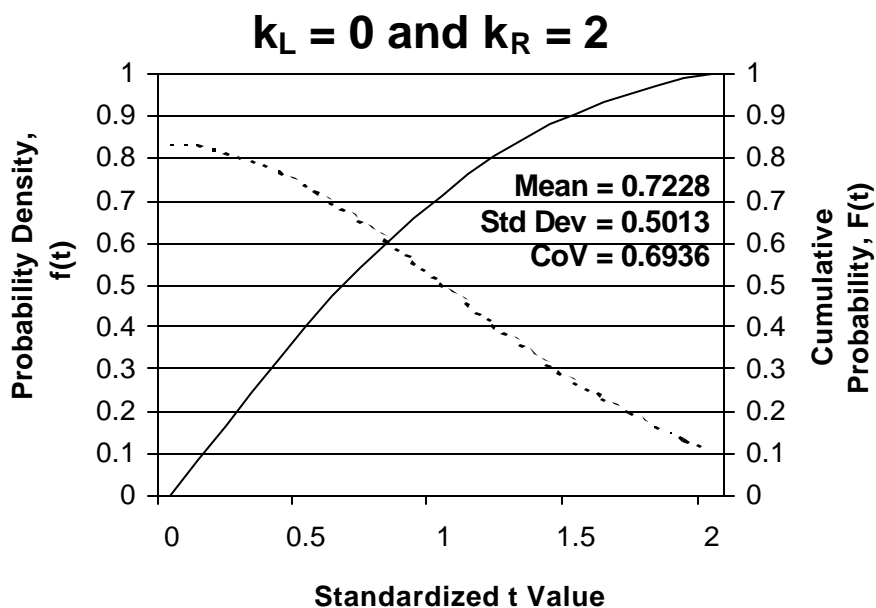


Figure 21. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = 0$ and $k_R = 2$

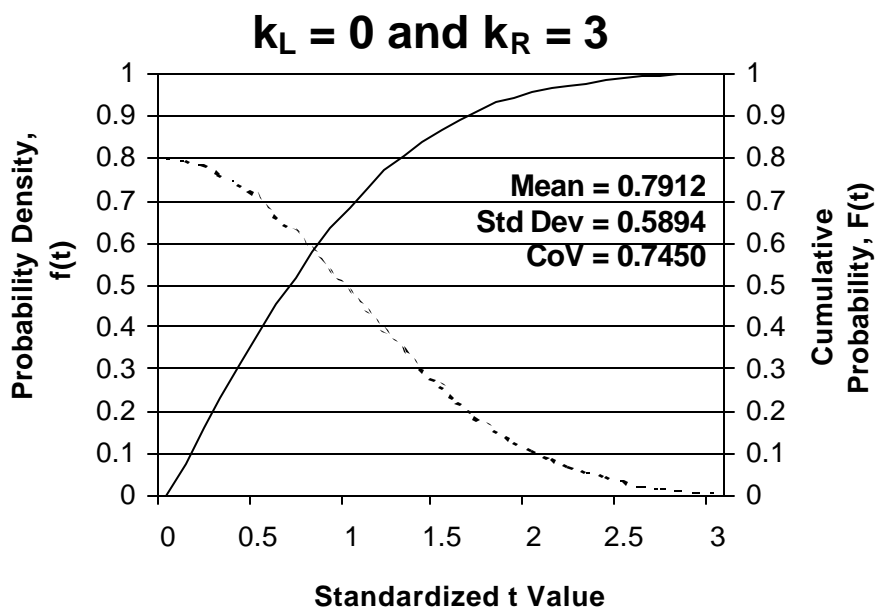


Figure 22. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = 0$ and $k_R = 3$

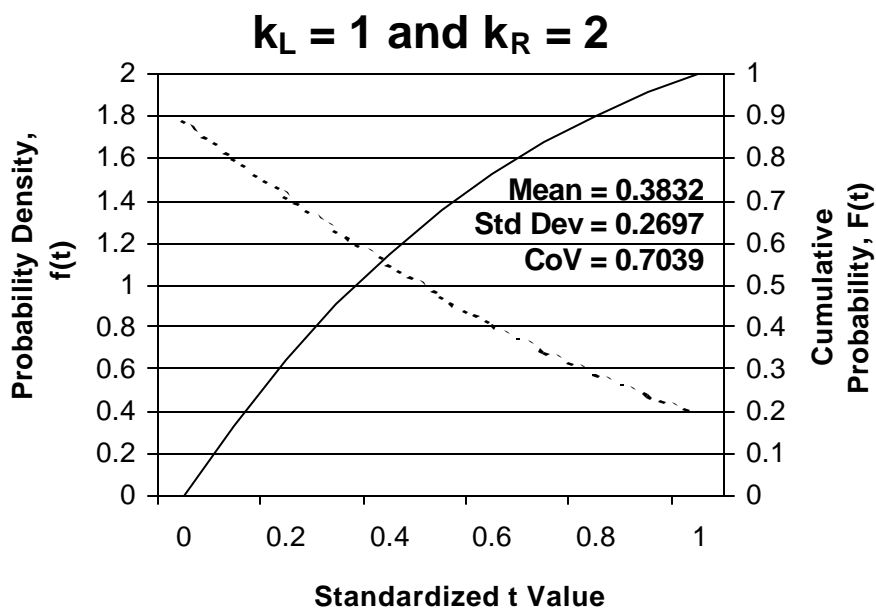


Figure 23. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = 1$ and $k_R = 2$

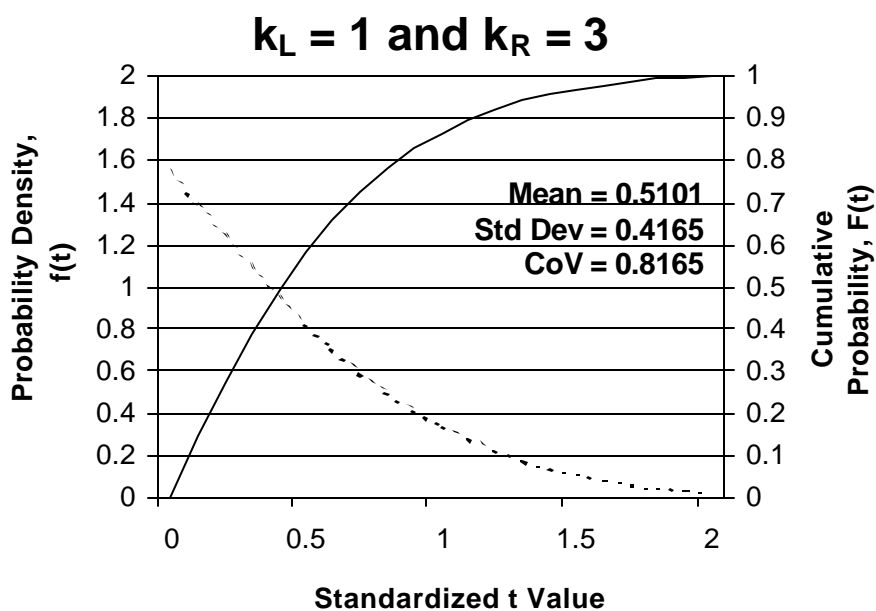


Figure 24. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = 1$ and $k_R = 3$

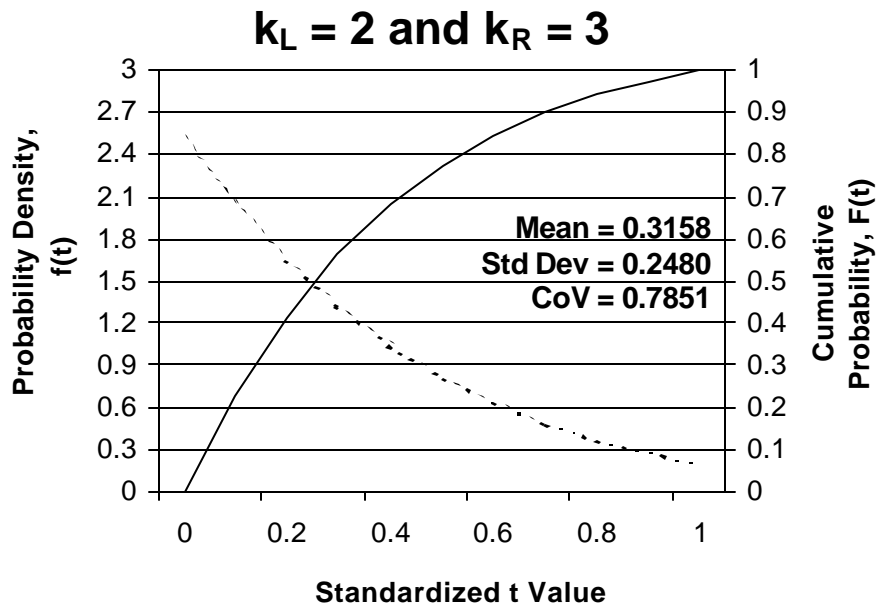


Figure 25. Visualization of the Standardized, Doubly-Truncated Normal Distribution with Points of Truncation $k_L = 2$ and $k_R = 3$

BIBLIOGRAPHY

- [Abr72] Abramowitz, M., and Stegun, I., Handbook of Mathematical Functions, New York: Dover Publications, 1972.
- [Ait58] Aitchison, J., and Silvey, S., "Maximum-Likelihood Estimation of Parameters Subject to Restraints," Annals of Mathematical Statistics, Vol. 29, pp. 813-828, 1958.
- [Bar99] Barr, D., and Sherrill, E., "Mean and Variance of Truncated Normal Distributions," The American Statistician, Vol. 53, pp. 357-361, 1999.
- [Bey87] Beyer, W. (ed.), CRC Standard Mathematical Tables, Boca Raton: CRC Press, 28th Edition, 1987.
- [Cas94] Castillo, J., "The Singly Truncated Normal Distribution: A Non-Steep Exponential Family," Annals of the Institute of Statistical Mathematics, Vol. 46, pp. 57-66, 1994.
- [Chi81] Chiu, W., and Leung, M., "A Graphical Method for Estimating the Parameters of a Truncated Normal Distribution," Journal of Quality Technology, Vol. 13, pp. 42-45, 1981.
- [Coh49] Cohen, A., "On Estimating the Mean and Standard Deviation of Truncated Normal Distributions," Journal of the American Statistical Association, Vol. 44, pp. 518-525, 1949.
- [Coh50a] Cohen, A., "Estimating Parameters of Pearson Type III Populations From Truncated Samples," Journal of the American Statistical Association, Vol. 45, pp. 411-423, 1950.
- [Coh50b] Cohen, A., "Estimating the Mean and Variance of Normal Populations From Singly Truncated and Doubly Truncated Samples," Annals of

Mathematical Statistics, Vol. 21, pp. 557-569, 1950.

- [Coh50c] Cohen, A., "On Estimating the Mean and Variance of Singly Truncated Normal Frequency Distributions From the First Three Sample Moments," Annals of the Institute of Statistical Mathematics (Tokyo), Vol. 3, pp. 37-44, 1950.
- [Coh52] Cohen, A., "Estimation of Parameters in Truncated Pearson Frequency Distributions," Annals of Mathematical Statistics, Vol. 22, pp. 256-265, 1952.
- [Coh53] Cohen, A., and Woodward, J., "Tables of Pearson-Lee-Fisher Functions of Singly Truncated Normal Distributions," Biometrics, Vol. 9, pp. 489-497, 1953.
- [Coh55a] Cohen, A., "Censored Samples from Truncated Normal Distributions," Biometrika, Vol. 42, pp. 516-519, 1955.
- [Coh55b] Cohen, A., "Restriction and Selection in Samples From Bivariate Normal Distributions," Journal of the American Statistical Association, Vol. 50, pp. 884-893, 1955.
- [Coh57a] Cohen, A., "On the Solution of Estimating Equations for Truncated and Censored Samples From Normal Populations," Biometrika, Vol. 44, pp. 225-236, 1957.
- [Coh57b] Cohen, A., "Restriction and Selection in Multinormal Distributions," Annals of Mathematical Statistics, Vol. 28, pp. 731-741, 1957.
- [Coh59] Cohen, A., "Simplified Estimators for the Normal Distribution When Samples Are Singly Censored or Truncated," Technometrics, Vol. 1, pp. 217-237, 1959.
- [Coh61a] Cohen, A., "Estimating the Poisson Parameters From Samples That Are Truncated on the

- Right," Technometrics, Vol. 3, pp. 433-438, 1961.
- [Coh61b] Cohen, A., "Tables for Maximum Likelihood Estimates: Singly Truncated and Singly Censored Samples," Technometrics, Vol. 3, pp. 535-541, 1961.
- [Coh91] Cohen, A., Truncated and Censored Samples, New York: Marcel Dekker, 1991.
- [Cra79] Crain, B., "Estimating the Parameters of a Truncated Normal Distribution," Applied Mathematics and Computation, Vol. 5, pp. 149-156, 1979.
- [Fis30] Fisher, R., "The Moments of the Distribution of Normal Samples of Measures of Departure From Normality," Proceedings of the Royal Society of London, Vol. 130, pp. 16-28, 1930.
- [Fis31] Fisher, R., "The Truncated Normal Distribution," British Association for the Advancement of Science, Mathematical Tables, Vol. 5, pp. xxxiii-xxxiv, 1931.
- [Gup52] Gupta, A., "Estimation of the Mean and Standard Deviation of a Normal Population From a Censored Sample," Biometrika, Vol. 39, pp. 260-273, 1952.
- [Hal49] Hald, A., "Maximum Likelihood Estimation of the Parameters of a Normal Distribution Which is Truncated at a Known Point," Skandinavisk Aktuarietidskrift, Vol. 32, pp. 119-134, 1949.
- [Hal52] Hald, A., Statistical Theory with Engineering Applications, New York: John Wiley and Sons, 1952.
- [Hal53] Halperin, M., "Estimation in the Truncated Normal Distribution," Journal of the American Statistical Association, Vol. 47, pp. 457-465, 1953.

- [Han80] Hansen, J., and Zeger, S., "The Asymptotic Variance of the Estimated Proportion Truncated From a Normal Population," Technometrics, Vol. 22, pp. 271-274, 1980.
- [Har66] Harter, H., and Moore, A., "Iterative Maximum Likelihood Estimation of the Parameters of Normal Populations From Singly and Doubly Censored Samples," Biometrika, Vol. 53, pp. 205-213, 1966.
- [Heg89] Hegde, L., and Dahiya, R., "Estimation of Parameters in a Truncated Normal Distribution," Communications in Statistics - Theory and Methods, Vol. 18, pp. 4177-4195, 1989.
- [Hin90] Hines, W., and Montgomery, D., Probability and Statistics in Engineering and Management Science, New York: John Wiley and Sons, 1990.
- [Joh70] Johnson, N., and Kotz, S., Distributions in Statistics: Continuous Univariate Distributions 1, New York: Houghton Mifflin, 1970.
- [Kec91] Kececioglu, D., Reliability Engineering Handbook, Vol. 1, Englewood Cliffs: Prentice Hall, 1991.
- [Kha79] Khatri, C., and Ratani, R., "On Estimation of the Mean Parameter of a Truncated Normal Distribution With Known Coefficient of Variation," Communications in Statistics - Theory and Methods, Vol. 8, pp. 237-244, 1979.
- [Law91] Law, A., and Kelton, W., Simulation Modeling and Analysis, New York: McGraw-Hill, 1991.
- [Mit87] Mittal, M., and Dahiya, R., "Estimating the Parameters of a Doubly Truncated Normal Distribution," Communications in Statistics - Simulation, Vol. 16, pp. 141-159, 1987.

- [Nor80] Norgaard, R., and Killeen, T., "Expected Utility and the Truncated Normal Distribution," Management Science, Vol. 26, pp. 901-909, 1980.
- [Pat82] Patel, J., and Read, C., Handbook of the Normal Distribution, New York: Marcel Dekker, 1982.
- [Pea55] Pearson, K., "The Normal Probability Function: Tables of Certain Area-Ordinate Ratios and of Their Reciprocals," Biometrika, Vol. 42, pp. 217-222, 1955.
- [Raj53] Raj, D., "Estimation of the Parameters of Type III Populations From Truncated Samples," Journal of the American Statistical Association, Vol. 48, pp. 336-349, 1953.
- [Rob64] Robson, D., and Whitlock, J., "Estimation of a Truncation Point," Biometrika, Vol. 51, pp. 33-39, 1964.
- [Sar56] Sarhan, A., and Greenberg, B., "Estimation of Location and Scale Parameters by Order Statistics From Single and Doubly Censored Samples," Annals of Mathematical Statistics, Vol. 27, pp. 427-451, 1956.
- [Sch86] Schneider, H., Truncated and Censored Samples From Normal Populations, New York: Marcel Dekker, 1986.
- [Sha66] Shah, S., and Jaiswal, M., "Estimation of Parameters of Doubly Truncated Normal Distribution From First Four Sample Moments," Annals of the Institute of Statistical Mathematics (Tokyo), Vol. 18, pp. 107-111, 1966.
- [Ste93] Stein, W., Pfaffenberger, R., and Mizzi, P., "A Stochastic Dominance Comparison of Truncated Normal Distributions," European Journal of Operational Research, Vol. 67, pp. 259-266, 1993.
- [Tho80] Thomopoulos, N., Applied Forecasting Methods, Englewood Cliffs: Prentice-Hall, 1980.